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Sixth Edition

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The late **MURRAY R. SPIEGEL** received his MS degree in Physics and PhD in Mathematics from Cornell University. He had positions at Harvard University, Columbia University, Oak Ridge and Rensselaer Polytechnic Institute, and served as a mathematical consultant at several large companies. His last position was professor and chairman of mathematics at Rensselaer Polytechnic Institute, Hartford Graduate Center. He was interested in most branches of mathematics, especially those involving applications to physics and engineering problems. He was the author of numerous journal articles and 14 books on various topics in mathematics.

LARRY J. STEPHENS is professor of mathematics at the University of Nebraska at Omaha, where he has taught since 1974. He has also taught at the University of Arizona, Gonzaga University, and Oklahoma State University. He has worked for NASA, Lawrence Berkeley National Laboratory, and Los Alamos National Laboratory. He has consulted widely, and spent 10 years as a consultant and conducted seminars for the engineering group at 3M in Valley, Nebraska. Dr. Stephens has over 35 years of experience teaching statistical methodology, engineering statistics, and mathematical statistics. He has over 50 publications in professional journals, and has written books in the Schaum's Outlines Series as well as books in the Utterly Confused and Demystified series published by McGraw-Hill Education.

To the memory of my Mother and Father, Rosie and Johnie Stephens

L.J.S.

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Preface to the Sixth Edition

Computer software has been included as part of *Schaum's Outline of Statistics* since the third edition of this book. However, with rapid development of computers and computer software since the publication of the third edition, the inclusion of computer software in the outline has increased steadily. I relied heavily on MINITAB, EXCEL, and STATISTIX in the sixth edition. The output from MINITAB and STATISTIX has helped clarify some of the statistical concepts which are hard to understand without some help from the graphics involved. I wish to thank the producers of the software packages for their permission to include output from their software in the book. I am indebted to Laura Brown, head of the author's assistance program at MINITAB, for all the help she has provided me as well as Dr. Gerard Nimis at STATISTIX for providing me with STATISTIX software to utilize as I was writing the sixth edition of the outline. Thanks to Diane Grayson at McGraw-Hill Education for the great work and time she has spent with me on the project. I also wish to thank my wife Lana, for all her help.

Larry Stephens,
Professor Emeritus at the University of Nebraska at Omaha

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Preface to the Fourth Edition

This fourth edition, completed in 2007, contains new examples, 130 new figures, and output from five computer software packages representative of the hundreds or perhaps thousands of computer software packages that are available for use in statistics. All figures in the third edition are replaced by new and sometimes different figures created by the five software packages: EXCEL, MINITAB, SAS, SPSS, and STATISTIX. My examples were greatly influenced by *USA Today* as I put the book together. This newspaper is a great source of current uses and examples in statistics.

Other changes in the book include the following. Chapter 18 on the analyses of time series was removed and Chapter 19 on statistical process control and process capability is now Chapter 18. The answers to the supplementary problems at the end of each chapter are given in more detail. More discussion and use of p-values are included throughout the book.

ACKNOWLEDGMENTS

As the statistical software plays such an important role in the book, I would like to thank the following people and companies for the right to use their software.

MINITAB: Ms. Laura Brown, Coordinator of the Author Assistance Program, Minitab Inc., 1829 Pine Hall Road, State College, PA 16801. I am a member of the author assistance program that Minitab sponsors. ‘‘Portions of the input and output contained in this publication/book are printed with the permission of Minitab Inc. All material remains the exclusive property and copyright of Minitab Inc. All rights reserved.’’ The Web address for Minitab is www.minitab.com.

SAS: Ms. Sandy Varner, Marketing Operations Manager, SAS Publishing, Cary, NC. ‘‘Created with SAS software. Copyright 2006. SAS Institute Inc., Cary, NC, USA. All rights reserved. Reproduced with permission of SAS Institute Inc., Cary, NC.’’ I quote from the Website: ‘‘SAS is the leader in business intelligence software and services. Over its 30 years, SAS has grown—from seven employees to nearly 10,000 worldwide, from a few customer sites to more than 40,000—and has been profitable every year.’’ The web address for SAS is www.sas.com.

SPSS: Ms. Jill Rietema, Account Manager, Publications, SPSS. I quote from the Website: ‘‘SPSS Inc. is a leading worldwide provider of predictive analytics software and solutions. Founded in 1968, today SPSS has more than 250,000 customers worldwide, served by more than 1200 employees in 60 countries.’’ The Web address for SPSS is www.spss.com.

STATISTIX: Dr. Gerard Nimis, President, Analytical Software, P.O. Box 12185, Tallahassee, FL 32317. I quote from the Website: ‘‘If you have data to analyze, but you’re a researcher, not a statistician, Statistix is designed for you. You’ll be up and running in minutes—without programming or using a manual. This easy to learn and simple to use software saves you valuable time and money. Statistix combines all the basic and advanced statistics and powerful data manipulation tools you need in a single, inexpensive package.’’ The Web address for STATISTIX is www.statistix.com.

EXCEL: Microsoft Excel has been around since 1985. It is available to almost all college students. It is widely used in this book.

I wish to thank Stanley Wileman for the computer advice that he so unselfishly gave to me during the writing of the book. I wish to thank my wife, Lana, for her understanding while I was often day dreaming and thinking about the best way to present some concept. Thanks to Chuck Wall, Senior Acquisitions Editor, and his staff at McGraw-Hill. Finally, I would like to thank Jeremy Toynbee, project manager at Keyword Publishing Services Ltd., London, England, and John Omiston, freelance copy editor, for their fine production work. I invite comments and questions about the book at Lstephens@mail.unomaha.edu.

LARRY J. STEPHENS

Preface to the Third Edition

In preparing this third edition of *Schaum's Outline of Statistics*, I have replaced dated problems with problems that reflect the technological and sociological changes that have occurred since the first edition was published in 1961. One problem in the second edition dealt with the lifetimes of radio tubes, for example. Since most people under 30 probably do not know what radio tubes are, this problem, as well as many other problems, have been replaced by problems involving current topics such as health care issues, AIDS, the Internet, cellular phones, and so forth. The mathematical and statistical aspects have not changed, but the areas of application and the computational aspects of statistics have changed.

Another important improvement is the introduction of statistical software into the text. The development of statistical software packages such as SAS, SPSS, and MINITAB has dramatically changed the application of statistics to real-world problems. One of the most widely used statistical packages in academia as well as in industrial settings is the package called MINITAB (Minitab Inc., 1829 Pine Hall Road, State College, PA 16801). I would like to thank Minitab Inc. for granting me permission to include MINITAB output throughout the text. Many modern statistical textbooks include computer software output as part of the text. I have chosen to include MINITAB because it is widely used and is very friendly. Once a student learns the various data file structures needed to use MINITAB, and the structure of the commands and subcommands, this knowledge is readily transferable to other statistical software. With the introduction of pull-down menus and dialog boxes, the software has been made even friendlier. I include both commands and pull-down menus in the MINITAB discussions in the text.

Many of the new problems discuss the important statistical concept of the *p*-value for a statistical test. When the first edition was introduced in 1961, the *p*-value was not as widely used as it is today, because it is often difficult to determine without the aid of computer software. Today *p*-values are routinely provided by statistical software packages since the computer software computation of *p*-values is often a trivial matter.

A new chapter entitled ''Statistical Process Control and Process Capability'' has replaced Chapter 19, ''Index Numbers.'' These topics have many industrial applications, and I felt they needed to be included in the text. The inclusion of the techniques of statistical process control and process capability in modern software packages has facilitated the implementation of these techniques in many industrial settings. The software performs all the computations, which are rather burdensome. I chose to use MINITAB because I feel that it is among the best software for SPC applications.

I wish to thank my wife Lana for her understanding during the preparation of the book; my friend Stanley Wileman for all the computer help he has given me; and Alan Hunt and the staff at Keyword Publishing Services Ltd., London, England, for their fine production work. Finally, I wish to thank the staff at McGraw-Hill for their cooperation and helpfulness.

LARRY J. STEPHENS

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Preface to the Second Edition

Statistics, or statistical methods as it is sometimes called, is playing an increasingly important role in nearly all phases of human endeavor. Formerly dealing only with affairs of the state, thus accounting for its name, the influence of statistics has now spread to agriculture, biology, business, chemistry, communications, economics, education, electronics, medicine, physics, political science, psychology, sociology, and numerous other fields of science and engineering.

The purpose of this book is to present an introduction to the general statistical principles which will be found useful to all individuals regardless of their fields of specialization. It has been designed for use either as a supplement to all current standard texts or as a textbook for a formal course in statistics. It should also be of considerable value as a book of reference for those presently engaged in applications of statistics to their own special problems of research.

Each chapter begins with clear statements of pertinent definitions, theorems, and principles together with illustrative and other descriptive material. This is followed by graded sets of solved and supplementary problems which in many instances use data drawn from actual statistical situations. The solved problems serve to illustrate and amplify the theory, bring into sharp focus those fine points without which the students continually feel themselves to be on unsafe ground, and provide the repetition of basic principles so vital to effective teaching. Numerous derivations of formulas are included among the solved problems. The large number of supplementary problems with answers serve as a complete review of the material of each chapter.

The only mathematical background needed for an understanding of the entire book is arithmetic and the elements of algebra. A review of important mathematical concepts used in the book is presented in the first chapter which may either be read at the beginning of the course or referred to later as the need arises.

The early part of the book deals with the analysis of frequency distributions and associated measures of central tendency, dispersion, skewness, and kurtosis. This leads quite naturally to a discussion of elementary probability theory and applications, which paves the way for a study of sampling theory. Techniques of large sampling theory, which involve the normal distribution, and applications to statistical estimation and tests of hypotheses and significance are treated first. Small sampling theory, involving student's t distribution, the chi-square distribution, and the F distribution together with the applications appear in a later chapter. Another chapter on curve fitting and the method of least squares leads logically to the topics of correlation and regression involving two variables. Multiple and partial correlation involving more than two variables are treated in a separate chapter. These are followed by chapters on the analysis of variance and nonparametric methods, new in this second edition. Two final chapters deal with the analysis of time series and index numbers respectively.

Considerably more material has been included here than can be covered in most first courses. This has been done to make the book more flexible, to provide a more useful book of reference, and to stimulate further interest in the topics. In using the book it is possible to change the order of many later chapters or even to omit certain chapters without difficulty. For example, Chapters 13–15 and 18–19 can, for the most part, be introduced immediately after Chapter 5, if it is desired to treat correlation, regression, times series, and index numbers before sampling theory. Similarly, most of Chapter 6 may be omitted if one does not wish to devote too much time to probability. In a first course all of Chapter 15 may be omitted. The present order has been used because there is an increasing tendency in modern courses to introduce sampling theory and statistical influence as early as possible.

I wish to thank the various agencies, both governmental and private, for their cooperation in supplying data for tables. Appropriate references to such sources are given throughout the book. In particular, I am indebted to Professor Sir Ronald A. Fisher, F.R.S., Cambridge; Dr. Frank Yates, F.R.S., Rothamsted; and Messrs. Oliver and Boyd Ltd., Edinburgh, for permission to use data from Table III of their book *Statistical Tables for Biological, Agricultural, and Medical Research*. I also wish to thank Esther and Meyer Scher for their encouragement and the staff of McGraw-Hill for their cooperation.

MURRAY R. SPIEGEL

Contents

CHAPTER 1

Variables and Graphs

STATISTICS

Statistics is concerned with scientific methods for collecting, organizing, summarizing, presenting, and analyzing data as well as with drawing valid conclusions and making reasonable decisions on the basis of such analysis.

In a narrower sense, the term *statistics* is used to denote the data themselves or numbers derived from the data, such as averages. Thus we speak of employment statistics, accident statistics, etc.

POPULATION AND SAMPLE; INDUCTIVE AND DESCRIPTIVE STATISTICS

In collecting data concerning the characteristics of a group of individuals or objects, such as the heights and weights of students in a university or the numbers of defective and nondefective bolts produced in a factory on a given day, it is often impossible or impractical to observe the entire group, especially if it is large. Instead of examining the entire group, called the *population*, or *universe*, one examines a small part of the group, called a *sample*.

A population can be *finite* or *infinite*. For example, the population consisting of all bolts produced in a factory on a given day is finite, whereas the population consisting of all possible outcomes (heads, tails) in successive tosses of a coin is infinite.

If a sample is representative of a population, important conclusions about the population can often be inferred from analysis of the sample. The phase of statistics dealing with conditions under which such inference is valid is called *inductive statistics*, or *statistical inference*. Because such inference cannot be absolutely certain, the language of *probability* is often used in stating conclusions.

The phase of statistics that seeks only to describe and analyze a given group without drawing any conclusions or inferences about a larger group is called *descriptive*, or *deductive*, *statistics*.

Before proceeding with the study of statistics, let us review some important mathematical concepts.

VARIABLES: DISCRETE AND CONTINUOUS

A *variable* is a symbol, such as X, Y, H, x, or B, that can assume any of a prescribed set of values, called the *domain* of the variable. If the variable can assume only one value, it is called a *constant*.

A variable that can theoretically assume any value between two given values is called a *continuous variable*; otherwise, it is called a *discrete variable*.

EXAMPLE 1. The number N of children in a family, which can assume any of the values 0, 1, 2, 3, … but cannot be 2.5 or 3.842, is a discrete variable.

EXAMPLE 2. The height H of an individual, which can be 62 inches (in), 63.8 in, or 65.8341 in, depending on the accuracy of measurement, is a continuous variable.

Data that can be described by a discrete or continuous variable are called *discrete data* or *continuous data*, respectively. The number of children in each of 1000 families is an example of discrete data, while the heights of 100 university students is an example of continuous data. In general, *measurements* give rise to continuous data, while *enumerations*, or *countings*, give rise to discrete data.

It is sometimes convenient to extend the concept of variables to nonnumerical entities; for example, color C in a rainbow is a variable that can take on the "values" red, orange, yellow, green, blue, indigo, and violet. It is generally possible to replace such variables by numerical quantities; for example, denote red by 1, orange by 2, etc.

ROUNDING OF DATA

The result of rounding a number such as 72.8 to the nearest unit is 73, since 72.8 is closer to 73 than to 72. Similarly, 72.8146 rounded to the nearest hundredth (or to two decimal places) is 72.81, since 72.8146 is closer to 72.81 than to 72.82.

In rounding 72.465 to the nearest hundredth, however, we are faced with a dilemma since 72.465 is *just as far* from 72.46 as from 72.47. It has become the practice in such cases to round to the *even integer* preceding the 5. Thus 72.465 is rounded to 72.46, 183.575 is rounded to 183.58, and 116,500,000 rounded to the nearest million is 116,000,000. This practice is especially useful in minimizing *cumulative rounding errors* when a large number of operations is involved (see Problem 1.4).

SCIENTIFIC NOTATION

When writing numbers, especially those involving many zeros before or after the decimal point, it is convenient to employ the scientific notation using powers of 10.

EXAMPLE 3. $10^1 = 10$, $10^2 = 10\times 10 = 100$, $10^5 = 10\times 10\times 10\times 10\times 10 = 100{,}000$, and $10^8 = 100{,}000{,}000$.

EXAMPLE 4. $10^0 = 1$; $10^{-1} = .1$, or 0.1; $10^{-2} = .01$, or 0.01; and $10^{-5} = .00001$, or 0.00001.

EXAMPLE 5. $864{,}000{,}000 = 8.64\times 10^8$, and $0.00003416 = 3.416\times 10^{-5}$.

Note that multiplying a number by 10^8, for example, has the effect of moving the decimal point of the number eight places *to the right*. Multiplying a number by 10^{-6} has the effect of moving the decimal point of the number six places *to the left*.

We often write 0.1253 rather than .1253 to emphasize the fact that a number other than zero before the decimal point has not accidentally been omitted. However, the zero before the decimal point can be omitted in cases where no confusion can result, such as in tables.

Often we use parentheses or dots to show the multiplication of two or more numbers. Thus $(5)(3) = 5\cdot 3 = 5\times 3 = 15$, and $(10)(10)(10) = 10\cdot 10\cdot 10 = 10\times 10\times 10 = 1000$. When letters are used to represent numbers, the parentheses or dots are often omitted; for example, $ab = (a)(b) = a\cdot b = a\times b$.

The scientific notation is often useful in computation, especially in locating decimal points. Use is then made of the rules

$$(10^p)(10^q) = 10^{p+q} \qquad \frac{10^p}{10^q} = 10^{p-q}$$

where p and q are any numbers.

In 10^p, p is called the *exponent* and 10 is called the *base*.

EXAMPLE 6. $(10^3)(10^2) = 1000\times 100 = 100{,}000 = 10^5 \qquad$ i.e., 10^{3+2}

$$\frac{10^6}{10^4} = \frac{1{,}000{,}000}{10{,}000} = 100 = 10^2 \qquad \text{i.e., } 10^{6-4}$$

EXAMPLE 7. $(4{,}000{,}000)(0.0000000002) = (4\times 10^6)(2\times 10^{-10}) = (4)(2)(10^6)(10^{-10}) = 8\times 10^{6-10}$

$$= 8\times 10^{-4} = 0.0008$$

EXAMPLE 8. $$\frac{(0.006)(80,000)}{0.04} = \frac{(6\times10^{-3})(8\times10^{4})}{4\times10^{-2}} = \frac{48\times10^{1}}{4\times10^{-2}} = \left(\frac{48}{4}\right)\times10^{1-(-2)}$$
$$= 12\times10^{3} = 12,000$$

SIGNIFICANT FIGURES

If a height is accurately recorded as 65.4 in, it means that the true height lies between 65.35 and 65.45 in. The accurate digits, apart from zeros needed to locate the decimal point, are called the *significant digits*, or *significant figures*, of the number.

EXAMPLE 9. 65.4 has three significant figures.

EXAMPLE 10. 4.5300 has five significant figures.

EXAMPLE 11. $.0018 = 0.0018 = 1.8\times10^{-3}$ has two significant figures.

EXAMPLE 12. $.001800 = 0.001800 = 1.800\times10^{-3}$ has four significant figures.

Numbers associated with enumerations (or countings), as opposed to measurements, are of course exact and so have an unlimited number of significant figures. In some of these cases, however, it may be difficult to decide which figures are significant without further information. For example, the number 186,000,000 may have 3, 4, …, 9 significant figures. If it is known to have five significant figures, it would be better to record the number as either 186.00 million or 1.8600×10^{8}.

COMPUTATIONS

In performing calculations involving multiplication, division, and the extraction of roots of numbers, the final result can have no more significant figures than the numbers with the fewest significant figures (see Problem 1.9).

EXAMPLE 13. $73.24\times4.52 = (73.24)(4.52) = 331$

EXAMPLE 14. $1.648/0.023 = 72$

EXAMPLE 15. $\sqrt{38.7} = 6.22$

EXAMPLE 16. $(8.416)(50) = 420.8$ (if 50 is exact)

In performing additions and subtractions of numbers, the final result can have no more significant figures after the decimal point than the numbers with the fewest significant figures after the decimal point (see Problem 1.10).

EXAMPLE 17. $3.16 + 2.7 = 5.9$

EXAMPLE 18. $83.42 - 72 = 11$

EXAMPLE 19. $47.816 - 25 = 22.816$ (if 25 is exact)

The above rule for addition and subtraction can be extended (see Problem 1.11).

FUNCTIONS

If to each value that a variable X can assume there corresponds one or more values of a variable Y, we say that Y is a *function* of X and write $Y = F(X)$ (read "Y equals F of X") to indicate this functional dependence. Other letters (G, ϕ, etc.) can be used instead of F.

The variable X is called the *independent variable* and Y is called the *dependent variable*.

If only one value of Y corresponds to each value of X, we call Y a *single-valued function* of X; otherwise, it is called a *multiple-valued function* of X.

EXAMPLE 20. The total population P of the United States is a function of the time t, and we write $P = F(t)$.

EXAMPLE 21. The stretch S of a vertical spring is a function of the weight W placed on the end of the spring. In symbols, $S = G(W)$.

The functional dependence (or correspondence) between variables is often depicted in a table. However, it can also be indicated by an equation connecting the variables, such as $Y = 2X - 3$, from which Y can be determined corresponding to various values of X.

If $Y = F(X)$, it is customary to let $F(3)$ denote "the value of Y when $X = 3$," to let $F(10)$ denote "the value of Y when $X = 10$," etc. Thus if $Y = F(X) = X^2$, then $F(3) = 3^2 = 9$ is the value of Y when $X = 3$.

The concept of function can be extended to two or more variables (see Problem 1.17).

RECTANGULAR COORDINATES

Figure 1-1 shows an EXCEL scatter plot for four points. The *scatter plot* is made up of two mutually perpendicular lines called the X and Y *axes*. The X axis is horizontal and the Y axis is vertical. The two axes meet at a point called the *origin*. The two lines divide the *XY plane* into four regions denoted by I, II, III, and IV and called the first, second, third, and fourth *quadrants*. Four points are shown plotted in Fig. 1-1. The point (2, 3) is in the first quadrant and is plotted by going 2 units to the right along the X axis from the origin and 3 units up from there. The point (–2.3, 4.5) is in the second quadrant and is plotted by going 2.3 units to the left along the X axis from the origin and then 4.5 units up from there. The point (–4, –3) is in the third quadrant and is plotted by going 4 units to the left of the origin along the X axis and then 3 units down from there. The point (3.5, –4) is in the fourth quadrant and is plotted by going 3.5 units to the right along the X axis and then 4 units down from there. The first number in a pair is called the *abscissa* of the point and the second number is called the *ordinate* of the point. The abscissa and the ordinate taken together are called the *coordinates* of the point.

By constructing a Z axis through the origin and perpendicular to the XY plane, we can easily extend the above ideas. In such cases the coordinates of a point would be denoted by (X, Y, Z).

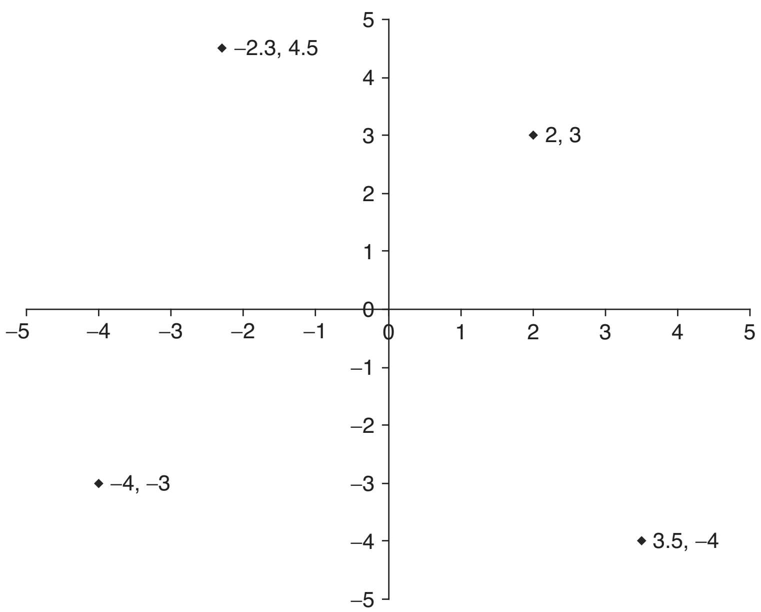

Fig. 1-1 An EXCEL plot of points in the four quadrants.

GRAPHS

A *graph* is a pictorial presentation of the relationship between variables. Many types of graphs are employed in statistics, depending on the nature of the data involved and the purpose for which the graph is intended. Among these are *bar graphs*, *pie graphs*, *pictographs*, etc. These graphs are sometimes referred to as *charts* or *diagrams*. Thus we speak of bar charts, pie diagrams, etc. (see Problems 1.23, 1.24, 1.25, 1.26, and 1.27).

Table 1.1 lists responses from owners of iPhones to the question: How did you damage your iPhone? See the Internet article *iPhone Damage Costs Americans Big Bucks* on HzO.com.

Table 1.1 Fifty causes for iPhone damage

Fell from hand	Immersed in liquid	Fell from lap	Immersed in liquid	Fell from lap
Fell from lap	Fell from hand	Fell from hand	Immersed in liquid	Fell from lap
Knocked off table	Liquid spilled on it	Fell from hand	Liquid spilled on it	Fell from hand
Immersed in liquid	Fell from lap	Fell from lap	Fell from hand	Fell from hand
Fell from lap	Immersed in liquid	Liquid spilled on it	Immersed in liquid	Fell from lap
Fell from hand	Liquid spilled on it	Immersed in liquid	Fell from lap	Fell from hand
Fell from lap	Immersed in liquid	Liquid spilled on it	Immersed in liquid	Liquid spilled on it
Knocked off table	Fell from lap	Fell from lap	Immersed in liquid	Immersed in liquid
Immersed in liquid	Knocked off table	Fell from lap	Fell from hand	Fell from lap
Liquid spilled on it	Immersed in liquid	Immersed in liquid	Knocked off table	Fell from hand

A *bar chart* is a plot of qualitative data in which the frequency or percent of the data is the length of a rectangle and each category is the base of the rectangle. Figure 1-2 is an EXCEL horizontal bar chart illustrating the data in Table 1.1.

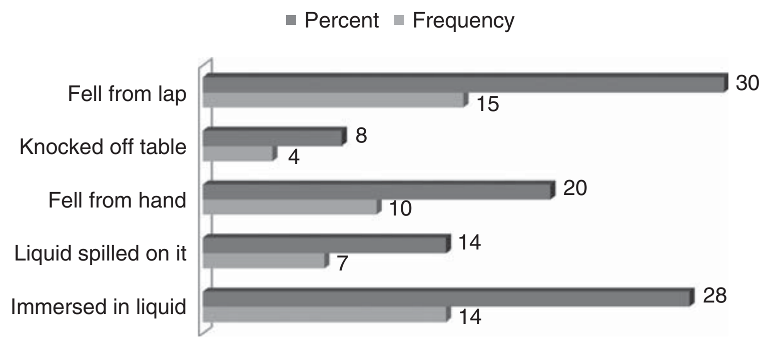

Fig. 1-2 EXCEL horizontal bar chart of data in Table 1.1.

A *pie chart* is a plot of qualitative data in which the size of the piece of pie is proportional to the frequency of a category. The angles of the pieces of pie that correspond to the categories add up to 360 degrees.

For example, the piece of pie that represents "Fell from lap" has an angle equal to 0.30(360 degrees) = 108 degrees. The piece of pie that represents "Knocked off table" has an angle equal to 0.08(360 degrees) = 28.8 degrees. The data in Table 1.1 may be summarized as follows.

Category	Frequency	Percent
Immersed in liquid	14	28
Liquid spilled on it	7	14
Fell from hand	10	20
Knocked off table	4	8
Fell from lap	15	30

EXCEL may be used to form a pie chart of the data. The data, in summarized form, is entered into the worksheet. EXCEL was used to form the following pie chart.

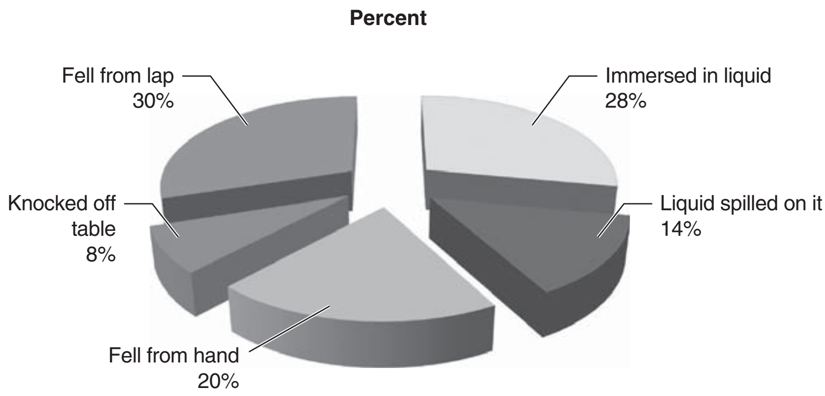

Fig. 1-3 EXCEL pie chart of data in Table 1.1.

EQUATIONS

Equations are statements of the form $A = B$, where A is called the *left-hand member* (or *side*) of the equation, and B the *right-hand member* (or *side*). So long as we apply the *same* operations to both members of an equation, we obtain *equivalent equations*. Thus we can add, subtract, multiply, or divide both members of an equation by the same value and obtain an equivalent equation, the only exception being that *division by zero is not allowed*.

EXAMPLE 22. Given the equation $2X+3=9$, subtract 3 from both members: $2X+3-3=9-3$, or $2X=6$. Divide both members by 2: $2X/2=6/2$, or $X = 3$. This value of X is a *solution* of the given equation, as seen by replacing X by 3, obtaining $2(3)+3=9$, or $9 = 9$, which is an *identity*. The process of obtaining solutions of an equation is called *solving* the equation.

The above ideas can be extended to finding solutions of two equations in two unknowns, three equations in three unknowns, etc. Such equations are called *simultaneous equations* (see Problem 1.29).

INEQUALITIES

The symbols $<$ and $>$ mean "less than" and "greater than," respectively. The symbols $\leq$ and $\geq$ mean "less than or equal to" and "greater than or equal to," respectively. They are known as *inequality symbols*.

EXAMPLE 23. $3 < 5$ is read "3 is less than 5."

EXAMPLE 24. $5 > 3$ is read "5 is greater than 3."

EXAMPLE 25. $X < 8$ is read "X is less than 8."

EXAMPLE 26. $X \geq 10$ is read "X is greater than or equal to 10."

EXAMPLE 27. $4 < Y \leq 6$ is read "4 is less than Y, which is less than or equal to 6," or Y is between 4 and 6, excluding 4 but including 6," or "Y is greater than 4 and less than or equal to 6."

Relations involving inequality symbols are called *inequalities*. Just as we speak of members of an equation, so we can speak of *members of an inequality*. Thus in the inequality $4 < Y \leq 6$, the members are 4, Y, and 6.

A valid inequality remains valid when:

1. The same number is added to or subtracted from each member.

EXAMPLE 28. Since 15 > 12, 15 + 3 > 12 + 3 (i.e., 18 > 15) and 15 – 3 > 12 – 3 (i.e., 12 > 9).

2. Each member is multiplied or divided by the same *positive* number.

EXAMPLE 29. Since 15 > 12, (15)(3) > (12)(3) (i.e., 45 > 36) and $15/3 > 12/3$ (i.e., 5 > 4).

3. Each member is multiplied or divided by the same *negative* number, provided that the inequality symbols are reversed.

EXAMPLE 30. Since 15 > 12, (15)(–3) < (12)(–3) (i.e., –45 < –36) and $15/(-3) < 12/(-3)$ (i.e., –5 < –4).

LOGARITHMS

For $x > 0$, $b > 0$, and $b \neq 1$, $y = \log_b x$ if and only if $\log b^y = x$. A logarithm is an exponent. It is the power that the base b must be raised to in order to get the number for which you are taking the logarithm. The two bases that have traditionally been used are 10 and e, which equals 2.71828182 . . . . Logarithms with base 10 are called *common logarithms* and are written as $\log_{10} x$ or simply as log(x). Logarithms with base e are called *natural logarithms* and are written as ln(x).

EXAMPLE 31. Find the following logarithms and then use EXCEL to find the same logarithms: $\log_2 8$, $\log_5 25$, and $\log_{10} 1000$. Three is the power of 2 that gives 8, and so $\log_2 8 = 3$. Two is the power of 5 that gives 25, and so $\log_5 25 = 2$. Three is the power of 10 that gives 1000, and so $\log_{10} 1000 = 3$. EXCEL contains three functions that compute logarithms. The function LN computes natural logarithms, the function LOG10 computes common logarithms, and the function LOG(x,b) computes the logarithm of x to base b. =LOG(8,2) gives 3, =LOG(25,5) gives 2, and =LOG10(1000) gives 3.

EXAMPLE 32. Use EXCEL to compute the natural logarithm of the integers 1 through 5. The numbers 1 through 5 are entered into B1:F1 and the expression =LN(B1) is entered into B2 and a click-and-drag is performed from B2 to F2. The following EXCEL output was obtained.

X	1	2	3	4	5
LN(x)	0	0.693147	1.098612	1.386294	1.609438

EXAMPLE 33. Show that the answers in Example 32 are correct by showing that $e^{\ln(x)}$ gives the value x. The logarithms are entered into B1:F1 and the expression $e^{\ln(x)}$, which is represented by =EXP(B1), is entered into B2 and a click-and-drag is performed from B2 to F2. The following EXCEL output was obtained. The numbers in D2 and E2 differ from 3 and 4 because of round off error.

LN(x)	0	0.693147	1.098612	1.386294	1.609438
x = EXP(LN(x))	1	2	2.999999	3.999999	5

Example 33 illustrates that if you have the logarithms of numbers ($\log_b(x)$) the numbers (x) may be recovered by using the relation $b^{\log_b(x)} = x$.

EXAMPLE 34. The number e may be defined as a limiting entity. The quantity $(1+(1/x))^x$ gets closer and closer to e when x grows larger. Consider the EXCEL evaluation of $(1+(1/x))^x$ for x = 1, 10, 100, 1000, 10000, 100000, and 1000000.

x	1	10	100	1000	10000	100000	1000000
(1 + 1/x)^x	2	2.593742	2.704814	2.716924	2.718146	2.718268	2.71828

The numbers 1, 10, 100, 1000, 10000, 100000, and 1000000 are entered into B1:H1 and the expression =(1+1/B1)^B1 is entered into B2 and a click-and-drag is performed from B2 to H2. This is expressed mathematically by the expression $\lim_{x\to\infty}(1+(1/x))^x = e$.

EXAMPLE 35. The balance of an account earning compound interest n times per year is given by $A(t) = P(1+(r/n))^{nt}$ where P is the principal, r is the interest rate, t is the time in years, and n is the number of compound periods per year. The balance of an account earning interest continuously is given by $A(t) = Pe^{rt}$. EXCEL is used to compare the continuous growth of $1000 and $1000 that is compounded quarterly after 1, 2, 3, 4, and 5 years at interest rate 5%. The results are:

Years	1	2	3	4	5
Quarterly	1050.95	1104.49	1160.75	1219.89	1282.04
Continuously	1051.27	1105.17	1161.83	1221.4	1284.03

The times 1, 2, 3, 4, and 5 are entered into B1:F1. The EXCEL expression =1000*(1.0125)^(4*B1) is entered into B2 and a click-and-drag is performed from B2 to F2. The expression =1000*EXP(0.05*B1) is entered into B3 and a click-and-drag is performed from B3 to F3. The continuous compounding produces slightly better results.

PROPERTIES OF LOGARITHMS

The following are the more important properties of logarithms:

1. $\log_b MN = \log_b M + \log_b N$
2. $\log_b M/N = \log_b M - \log_b N$
3. $\log_b M^P = p \log_b M$

EXAMPLE 36. Write $\log_b(xy^4/z^3)$ as the sum or difference of logarithms of x, y, and z.

$$\log_b \frac{xy^4}{z^3} = \log_b xy^4 - \log_b z^3 \qquad \text{property 2}$$

$$\log_b \frac{xy^4}{z^3} = \log_b x + \log_b y^4 - \log_b z^3 \qquad \text{property 1}$$

$$\log_b \frac{xy^4}{z^3} = \log_b x + 4\log_b y - 3\log_b z \qquad \text{property 3}$$

LOGARITHMIC EQUATIONS

To solve logarithmic equations:

1. Isolate the logarithms on one side of the equation.
2. Express a sum or difference of logarithms as a single logarithm.
3. Re-express the equation in step 2 in exponential form.
4. Solve the equation in step 3.
5. Check all solutions.

EXAMPLE 37. Solve the following logarithmic equation: $\log_4(x + 5) = 3$. First, re-express in exponential form as $x+5=4^3=64$. Then solve for x as follows. $x=64-5=59$. Then check your solution. $\log_4(59+5) = \log_4(64) = 3$ since $4^3 = 64$.

EXAMPLE 38. Solve the following logarithmic equation. $\log(6y-7)+\log y = \log(5)$. Replace the sum of logs by the log of the products. $\log(6y-7)y = \log(5)$. Now equate $(6y-7)y$ and 5. The result is $6y^2-7y=5$ or $6y^2-7y-5=0$. This quadratic equation factors as $(3y-5)(2y+1)=0$. The solutions are $y=5/3$ and $y=-1/2$. The $-1/2$ is rejected since it gives logs of negative numbers which are not defined. The $y=5/3$ checks out as a solution when tried in the original equation. Therefore our only solution is $y=5/3$.

EXAMPLE 39. Solve the following logarithmic equation:

$$\ln(5x) - \ln(4x+2) = 4.$$

Replace the difference of logs by the log of the quotient, $\ln(5x/(4x+2)) = 4$. Apply the definition of a logarithm: $5x/(4x+2) = e^4 = 54.59815$. Solving the equation $5x = 218.39260x + 109.19630$ for x gives $x = -0.5117$. However, this answer does not check in the equation $\ln(5x) - \ln(4x+2) = 4$, since the log function is not defined for negative numbers. The equation $\ln(5x) - \ln(4x+2) = 4$ has no solutions.

SOLVED PROBLEMS

Variables

1.1 State which of the following represent discrete data and which represent continuous data:

(*a*) Numbers of shares sold each day in the stock market

(*b*) Temperatures recorded every half hour at a weather bureau

(*c*) Lifetimes of television tubes produced by a company

(*d*) Yearly incomes of college professors

(*e*) Lengths of 1000 bolts produced in a factory

SOLUTION

(*a*) Discrete; (*b*) continuous; (*c*) continuous; (*d*) discrete; (*e*) continuous.

1.2 Give the domain of each of the following variables, and state whether the variables are continuous or discrete:

(*a*) Number G of gallons (gal) of water in a washing machine

(*b*) Number B of books on a library shelf

(*c*) Sum S of points obtained in tossing a pair of dice

(*d*) Diameter D of a sphere

(*e*) Country C in Europe

SOLUTION

(*a*) *Domain:* Any value from 0 gal to the capacity of the machine. *Variable:* Continuous.

(*b*) *Domain:* 0, 1, 2, 3, … up to the largest number of books that can fit on a shelf. *Variable:* Discrete.

(*c*) *Domain:* Points obtained on a single die can be 1, 2, 3, 4, 5, or 6. Hence the sum of points on a pair of dice can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12, which is the domain of S. *Variable:* Discrete.

(*d*) *Domain:* If we consider a point as a sphere of zero diameter, the domain of D is all values from zero upward. *Variable:* Continuous.

(*e*) *Domain:* England, France, Germany, etc., which can be represented numerically by 1, 2, 3, etc. *Variable:* Discrete.

Rounding of Data

1.3 Round each of the following numbers to the indicated accuracy:

(*a*)	48.6	nearest unit
(*b*)	136.5	nearest unit
(*c*)	2.484	nearest hundredth
(*d*)	0.0435	nearest thousandth

(*e*) 4.50001 nearest unit

(*f*) 143.95 nearest tenth

(*g*) 368 nearest hundred

(*h*) 24,448 nearest thousand

(*i*) 5.56500 nearest hundredth

(*j*) 5.56501 nearest hundredth

SOLUTION

(*a*) 49; (*b*) 136; (*c*) 2.48; (*d*) 0.044; (*e*) 5; (*f*) 144.0; (*g*) 400; (*h*) 24,000; (*i*) 5.56; (*j*) 5.57.

1.4 Add the numbers 4.35, 8.65, 2.95, 12.45, 6.65, 7.55, and 9.75 (*a*) directly, (*b*) by rounding to the nearest tenth according to the "even integer" convention, and (*c*) by rounding so as to increase the digit before the 5.

SOLUTION

(*a*)		(*b*)		(*c*)	
	4.35		4.4		4.4
	8.65		8.6		8.7
	2.95		3.0		3.0
	12.45		12.4		12.5
	6.65		6.6		6.7
	7.55		7.6		7.6
	9.75		9.8		9.8
Total	52.35	Total	52.4	Total	52.7

Note that procedure (*b*) is superior to procedure (*c*) because *cumulative rounding errors* are minimized in procedure (*b*).

Scientific Notation and Significant Figures

1.5 Express each of the following numbers without using powers of 10:

(*a*) 4.823×10^7 (*c*) 3.80×10^{-4} (*e*) 300×10^8

(*b*) 8.4×10^{-6} (*d*) 1.86×10^5 (*f*) $70{,}000 \times 10^{-10}$

SOLUTION

(*a*) Move the decimal point seven places to the right and obtain 48,230,000;

(*b*) move the decimal point six places to the left and obtain 0.0000084;

(*c*) 0.000380;

(*d*) 186,000;

(*e*) 30,000,000,000;

(*f*) 0.0000070000.

1.6 How many significant figures are in each of the following, assuming that the numbers are recorded accurately?

(*a*) 149.8 in (*d*) 0.00280 m (*g*) 9 houses

(*b*) 149.80 in (*e*) 1.00280 m (*h*) 4.0×10^3 pounds (lb)

(*c*) 0.0028 meter (m) (*f*) 9 grams (g) (*i*) 7.58400×10^{-5} dyne

SOLUTION

(*a*) Four; (*b*) five; (*c*) two; (*d*) three; (*e*) six; (*f*) one; (*g*) unlimited; (*h*) two; (*i*) six.

1.7 What is the maximum error in each of the following measurements, assuming that they are recorded accurately?

(*a*) 73.854 in (*b*) 0.09800 cubic feet (ft^3) (*c*) 3.867×10^8 kilometers (km)

SOLUTION

(*a*) The measurement can range anywhere from 73.8535 to 73.8545 in; hence the maximum error is 0.0005 in. Five significant figures are present.

(*b*) The number of cubic feet can range anywhere from 0.097995 to 0.098005 ft^3; hence the maximum error is 0.000005 ft^3. Four significant figures are present.

(*c*) The actual number of kilometers is greater than 3.8665×10^8 but less than 3.8675×10^8; hence the maximum error is 0.0005×10^8, or 50,000 km. Four significant figures are present.

1.8 Write each number using the scientific notation. Unless otherwise indicated, assume that all figures are significant.

(*a*) 24,380,000 (four significant figures)
(*b*) 0.000009851
(*c*) 7,300,000,000 (five significant figures)
(*d*) 0.00018400

SOLUTION

(*a*) 2.438×10^7; (*b*) 9.851×10^{-6}; (*c*) 7.3000×10^9; (*d*) 1.8400×10^{-4}.

Computations

1.9 Show that the product of the numbers 5.74 and 3.8, assumed to have three and two significant figures, respectively, cannot be accurate to more than two significant figures.

SOLUTION

First method

$5.74 \times 3.8 = 21.812$, but not all figures in this product are significant. To determine how many figures are significant, observe that 5.74 stands for any number between 5.735 and 5.745, while 3.8 stands for any number between 3.75 and 3.85. Thus the smallest possible value of the product is $5.735 \times 3.75 = 21.50625$, and the largest possible value is $5.745 \times 3.85 = 22.11825$.

Since the possible range of values is 21.50625 to 22.11825, it is clear that no more than the first two figures in the product can be significant, the result being written as 22. Note that the number 22 stands for any number between 21.5 and 22.5.

Second method

With doubtful figures in italic, the product can be computed as shown here:

$$\begin{array}{r} 5.7\mathit{4} \\ 3\mathit{8} \\ \hline \mathit{4592} \\ 172\mathit{2} \\ \hline 2\mathit{1.812} \end{array}$$

We should keep no more than one doubtful figure in the answer, which is therefore 22 to two significant figures. Note that it is unnecessary to carry more significant figures than are present in the least accurate factor; thus if 5.74 is rounded to 5.7, the product is $5.7 \times 3.8 = 21.66 = 22$ to two significant figures, agreeing with the above results.

In calculating without the use of computers, labor can be saved by not keeping more than one or two figures beyond that of the least accurate factor and rounding to the proper number of significant figures in the final answer. With computers, which can supply many digits, we must be careful not to believe that all the digits are significant.

1.10 Add the numbers 4.19355, 15.28, 5.9561, 12.3, and 8.472, assuming all figures to be significant.

SOLUTION

In calculation (*a*) below, the doubtful figures in the addition are in italic type. The final answer with no more than one doubtful figure is recorded as 46.2.

$$(a)\quad \begin{array}{r} 4.1935\mathit{5} \\ 15.2\mathit{8} \\ 5.956\mathit{1} \\ 12.\mathit{3} \\ 8.47\mathit{2} \\ \hline 46.\mathit{20165} \end{array} \qquad\qquad (b)\quad \begin{array}{r} 4.19 \\ 15.28 \\ 5.96 \\ 12.3 \\ 8.47 \\ \hline 46.20 \end{array}$$

Some labor can be saved by proceeding as in calculation (*b*), where we have kept one more significant decimal place than that in the least accurate number. The final answer, rounded to 46.2, agrees with calculation (*a*).

1.11 Calculate 475,000,000 + 12,684,000 − 1,372,410 if these numbers have three, five, and seven significant figures, respectively.

SOLUTION

In calculation (*a*) below, all figures are kept and the final answer is rounded. In calculation (*b*), a method similar to that of Problem 1.10(*b*) is used. In both cases, doubtful figures are in italic type.

$$(a)\quad \begin{array}{r} 47\mathit{5{,}000{,}000} \\ +\ 12{,}68\mathit{4{,}000} \\ \hline 487{,}\mathit{684{,}000} \end{array} \qquad \begin{array}{r} 487{,}\mathit{684{,}000} \\ -\ 1{,}372{,}41\mathit{0} \\ \hline 486{,}\mathit{311{,}590} \end{array} \qquad\qquad (b)\quad \begin{array}{r} 47\mathit{5{,}000{,}000} \\ +\ 12{,}\mathit{700{,}000} \\ \hline 487{,}\mathit{700{,}000} \end{array} \qquad \begin{array}{r} 487{,}\mathit{700{,}000} \\ -\ 1{,}400{,}\mathit{000} \\ \hline 486{,}\mathit{300{,}000} \end{array}$$

The final result is rounded to 486,000,000; or better yet, to show that there are three significant figures, it is written as 486 million or 4.86×10^8.

1.12 Perform each of the indicated operations.

(*a*) 48.0×943

(*b*) $8.35/98$

(*c*) $(28)(4193)(182)$

(*d*) $\dfrac{(526.7)(0.001280)}{0.000034921}$

(*e*) $\dfrac{(1.47562-1.47322)(4895.36)}{0.000159180}$

(*f*) If denominators 5 and 6 are exact, $\dfrac{(4.38)^2}{5}+\dfrac{(5.482)^2}{6}$

(*g*) $3.1416\sqrt{71.35}$

(*h*) $\sqrt{128.5-89.24}$

SOLUTION

(*a*) $48.0 \times 943 = (48.0)(943) = 45{,}300$

(*b*) $8.35/98 = 0.085$

(*c*) $$(28)(4193)(182) = (2.8\times10^1)(4.193\times10^3)(1.82\times10^2)$$
$$= (2.8)(4.193)(1.82)\times10^{1+3+2} = 21\times10^6 = 2.1\times10^7$$

This can also be written as 21 million to show the two significant figures.

(*d*) $$\frac{(526.7)(0.001280)}{0.000034921} = \frac{(5.267\times10^2)(1.280\times10^{-3})}{3.4921\times10^{-5}} = \frac{(5.267)(1.280)}{3.4921}\times\frac{(10^2)(10^{-3})}{10^{-5}}$$
$$= 1.931\times\frac{10^{2-3}}{10^{-5}} = 1.931\times\frac{10^{-1}}{10^{-5}}$$
$$= 1.931\times10^{-1+5} = 1.931\times10^4$$

This can also be written as 19.31 thousand to show the four significant figures.

(*e*) $$\frac{(1.47562-1.47322)(4895.36)}{0.000159180}=\frac{(0.00240)(4895.36)}{0.000159180}=\frac{(2.40\times10^{-3})(4.89536\times10^{3})}{1.59180\times10^{-4}}$$
$$=\frac{(2.40)(4.89536)}{1.59180}\times\frac{(10^{-3})(10^{3})}{10^{-4}}=7.38\times\frac{10^{0}}{10^{-4}}=7.38\times10^{4}$$

This can also be written as 73.8 thousand to show the three significant figures. Note that although six significant figures were originally present in all numbers, some of these were lost in subtracting 1.47322 from 1.47562.

(*f*) If denominators 5 and 6 are exact, $\dfrac{(4.38)^2}{5}+\dfrac{(5.482)^2}{6}=3.84+5.009=8.85$.

(*g*) $3.1416\sqrt{71.35}=(3.1416)(8.447)=26.54$

(*h*) $\sqrt{128.5-89.24}=\sqrt{39.3}=6.27$

1.13 Evaluate each of the following, given that $X=3$, $Y=-5$, $A=4$, and $B=-7$, where all numbers are assumed to be exact:

(*a*) $2X-3Y$

(*b*) $4Y-8X+28$

(*c*) $\dfrac{AX+BY}{BX-AY}$

(*d*) $X^2-3XY-2Y^2$

(*e*) $2(X+3Y)-4(3X-2Y)$

(*f*) $\dfrac{X^2-Y^2}{A^2-B^2+1}$

(*g*) $\sqrt{2X^2-Y^2-3A^2+4B^2+3}$

(*h*) $\sqrt{\dfrac{6A^2}{X}+\dfrac{2B^2}{Y}}$

SOLUTION

(*a*) $2X-3Y=2(3)-3(-5)=6+15=21$

(*b*) $4Y-8X+28=4(-5)-8(3)+28=-20-24+28=-16$

(*c*) $$\frac{AX+BY}{BX-AY}=\frac{(4)(3)+(-7)(-5)}{(-7)(3)-(4)(-5)}=\frac{12+35}{-21+20}=\frac{47}{-1}=-47$$

(*d*) $X^2-3XY-2Y^2=(3)^2-3(3)(-5)-2(-5)^2=9+45-50=4$

(*e*) $$2(X+3Y)-4(3X-2Y)=2[(3)+3(-5)]-4[3(3)-2(-5)]$$
$$=2(3-15)-4(9+10)=2(-12)-4(19)=-24-76=-100$$

Another method

$$2(X+3Y)-4(3X-2Y)=2X+6Y-12X+8Y=-10X+14Y=-10(3)+14(-5)$$
$$=-30-70=-100$$

(*f*) $$\frac{X^2-Y^2}{A^2-B^2+1}=\frac{(3)^2-(-5)^2}{(4)^2-(-7)^2+1}=\frac{9-25}{16-49+1}=\frac{-16}{-32}=\frac{1}{2}=0.5$$

(*g*) $$\sqrt{2X^2-Y^2-3A^2+4B^2+3}=\sqrt{2(3)^2-(-5)^2-3(4)^2+4(-7)^2+3}$$
$$=\sqrt{18-25-48+196+3}=\sqrt{144}=12$$

(*h*) $$\sqrt{\frac{6A^2}{X}+\frac{2B^2}{Y}}=\sqrt{\frac{6(4)^2}{3}+\frac{2(-7)^2}{-5}}=\sqrt{\frac{96}{3}+\frac{98}{-5}}=\sqrt{12.4}=3.52\qquad\text{approximately}$$

Functions and Graphs

1.14 Table 1.2 shows the number of bushels (bu) of wheat and corn produced on the Tyson farm during the years 2002, 2003, 2004, 2005, and 2006. With reference to this table, determine the year or years during which (*a*) the least number of bushels of wheat was produced, (*b*) the greatest number of bushels of corn was produced, (*c*) the greatest decline in wheat production occurred, (*d*) equal amounts of wheat were produced, and (*e*) the combined production of wheat and corn was a maximum.

Table 1.2 Wheat and corn production from 2002 to 2006

Year	Bushels of Wheat	Bushels of Corn
2002	205	80
2003	215	105
2004	190	110
2005	205	115
2006	225	120

SOLUTION

(*a*) 2004; (*b*) 2006; (*c*) 2004; (*d*) 2002 and 2005; (*e*) 2006

1.15 Let W and C denote, respectively, the number of bushels of wheat and corn produced during the year t on the Tyson farm of Problem 1.14. It is clear W and C are both functions of t; this we can indicate by $W = F(t)$ and $C = G(t)$.

(*a*) Find W when $t = 2004$.
(*b*) Find C when $t = 2002$.
(*c*) Find t when $W = 205$.
(*d*) Find $F(2005)$.
(*e*) Find $G(2005)$.
(*f*) Find C when $W = 190$.
(*g*) What is the domain of the variable t?
(*h*) Is W a single-valued function of t?
(*i*) Is t a function of W?
(*j*) Is C a function of W?
(*k*) Which variable is independent, t or W?

SOLUTION

(*a*) 190
(*b*) 80
(*c*) 2002 and 2005
(*d*) 205
(*e*) 115
(*f*) 110
(*g*) The years 2002 through 2006.
(*h*) Yes, since to each value that t can assume, there corresponds one and only one value of W.
(*i*) Yes, the functional dependence of t on W can be written $t = H(W)$.
(*j*) Yes.
(*k*) Physically, it is customary to think of W as determined from t rather than t as determined from W. Thus, physically, t is the dependent variable and W is the independent variable. Mathematically, however, either variable can be considered the independent variable and the other the dependent. The one that is assigned various values is the independent variable; the one that is then determined as a result is the dependent variable.

1.16 A variable Y is determined from a variable X according to the equation $Y = 2X - 3$, where the 2 and 3 are exact.

(*a*) Find Y when $X = 3, -2$, and 1.5.
(*b*) Construct a table showing the values of Y corresponding to $X = -2, -1, 0, 1, 2, 3$, and 4.
(*c*) If the dependence of Y on X is denoted by $Y = F(X)$, determine $F(2.4)$ and $F(0.8)$.
(*d*) What value of X corresponds to $Y = 15$?
(*e*) Can X be expressed as a function of Y?
(*f*) Is Y a single-valued function of X?
(*g*) Is X a single-valued function of Y?

SOLUTION

(*a*) When $X = 3$, $Y = 2X - 3 = 2(3) - 3 = 6 - 3 = 3$. When $X = -2$, $Y = 2X - 3 = 2(-2) - 3 = -4 - 3 = -7$. When $X = 1.5$, $Y = 2X - 3 = 2(1.5) - 3 = 3 - 3 = 0$.

(*b*) The values of Y, computed as in part (*a*), are shown in Table 1.3. Note that by using other values of X, we can construct many tables. The relation $Y = 2X - 3$ is equivalent to the collection of all such possible tables.

Table 1.3

X	−2	−1	0	1	2	3	4
Y	−7	−5	−3	−1	1	3	5

(*c*) $F(2.4) = 2(2.4) - 3 = 4.8 - 3 = 1.8$, and $F(0.8) = 2(0.8) - 3 = 1.6 - 3 = -1.4$.

(*d*) Substitute $Y = 15$ in $Y = 2X - 3$. This gives $15 = 2X - 3$, $2X = 18$, and $X = 9$.

(*e*) Yes. Since $Y = 2X - 3$, $Y + 3 = 2X$ and $X = \frac{1}{2}(Y+3)$. This expresses X *explicitly* as a function of Y.

(*f*) Yes, since for each value that X can assume (and there is an indefinite number of these values) there corresponds one and only one value of Y.

(*g*) Yes, since from part (*e*), $X = \frac{1}{2}(Y+3)$, so that corresponding to each value assumed by Y there is one and only one value of X.

1.17 If $Z = 16 + 4X - 3Y$, find the value of Z corresponding to (*a*) $X = 2$, $Y = 5$; (*b*) $X = -3$, $Y = -7$; and (*c*) $X = -4$, $Y = 2$.

SOLUTION

(*a*) $Z = 16 + 4(2) - 3(5) = 16 + 8 - 15 = 9$

(*b*) $Z = 16 + 4(-3) - 3(-7) = 16 - 12 + 21 = 25$

(*c*) $Z = 16 + 4(-4) - 3(2) = 16 - 16 - 6 = -6$

Given values of X and Y, there corresponds a value of Z. We can denote this dependence of Z on X and Y by writing $Z = F(X, Y)$ (read "Z is a function of X and Y"). $F(2, 5)$ denotes the value of Z when $X = 2$ and $Y = 5$ and is 9, from part (*a*). Similarly, $F(-3, -7) = 25$ and $F(-4, 2) = -6$ from parts (*b*) and (*c*), respectively.

The variables X and Y are called *independent variables*, and Z is called a *dependent variable*.

1.18 The overhead cost for a company is \$1000 per day and the cost of producing each item is \$25.

(*a*) Write a function that gives the total cost of producing x units per day.

(*b*) Use EXCEL to produce a table that evaluates the total cost for producing 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50 units a day.

(*c*) Evaluate and interpret $f(100)$.

SOLUTION

(*a*) $f(x) = 1000 + 25x$

(*b*) The numbers 5, 10, …, 50 are entered into B1:K1 and the expression =1000 + 25*B1 is entered into B2 and a click-and-drag is performed from B2 to K2 to form the following output.

x	5	10	15	20	25	30	35	40	45	50
f(x)	1025	1050	1075	1100	1125	1150	1175	1200	1225	1250

(*c*) $f(100) = 1000 + 25(100) = 1000 + 2500 = 3500$. The cost of manufacturing $x = 100$ units a day is 3500.

1.19 A rectangle has width x and length $x + 10$.

(*a*) Write a function, $A(x)$, that expresses the area as a function of x.

(*b*) Use EXCEL to produce a table that evaluates $A(x)$ for $x = 0, 1, \ldots, 5$.

(*c*) Write a function, $P(x)$, that expresses the perimeter as a function of x.

(*d*) Use EXCEL to produce a table that evaluates $P(x)$ for $x = 0, 1, \ldots, 5$.

SOLUTION

(*a*) $A(x) = x(x+10) = x^2 + 10x$

(*b*) The numbers 0, 1, 2, 3, 4, and 5 are entered into B1:G1 and the expression =B1^2 + 10*B1 is entered into B2 and a click-and-drag is performed from B2 to G2 to form the output:

X	0	1	2	3	4	5
A(x)	0	11	24	39	56	75

(*c*) $P(x) = x + (x+10) + x + (x+10) = 4x + 20.$

(*d*) The numbers 0, 1, 2, 3, 4, and 5 are entered into B1:G1 and the expression =4*B1 + 20 is entered into B2 and a click-and-drag is performed from B2 to G2 to form the output:

X	0	1	2	3	4	5
P(x)	20	24	28	32	36	40

1.20 Locate on a rectangular coordinate system the points having coordinates (*a*) (5, 2), (*b*) (2, 5), (*c*) (–5, 1), (*d*) (1, –3), (*e*) (3, –4), (*f*) (–2.5, –4.8), (*g*) (0, –2.5), and (*h*) (4, 0). Use MAPLE to plot the points.

SOLUTION

See Fig. 1-4. The MAPLE command to plot the eight points is given. Each point is represented by a circle.
L: = [[5, 2], [2, 5], [–5, 1], [1, –3], [3, –4], [–2.5, –4.8], [0, –2.5], [4, 0]]; *pointplot* (*L*, *font* = [*TIMES*, *BOLD*, *14*], *symbol* = *circle*);

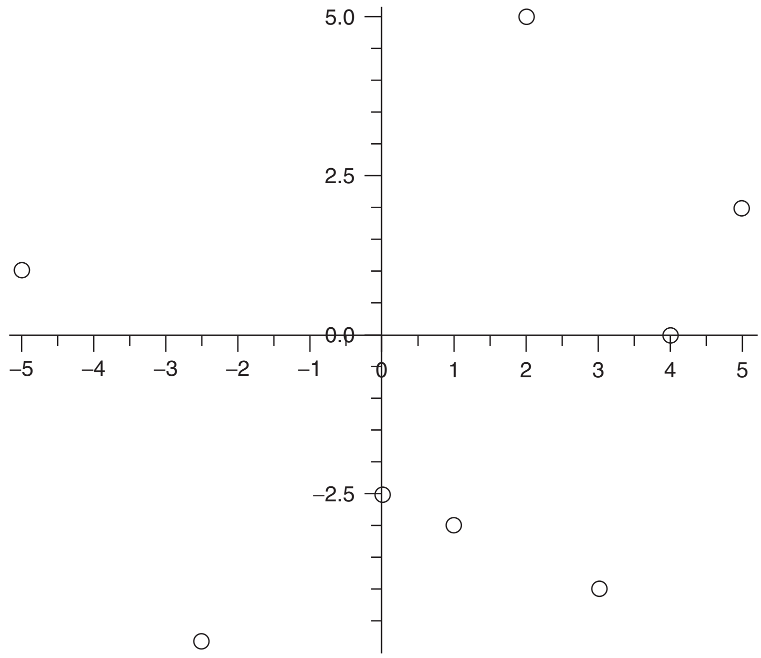

Fig. 1-4 A MAPLE plot of points.

1.21 Graph the equation $Y = 4X - 4$ using MINITAB.

SOLUTION

Note that the graph will extend indefinitely in the positive direction and negative direction of the *x* axis. We arbitrarily decide to go from –5 to 5. Figure 1-5 shows the MINITAB plot of the line $Y = 4X - 4$. The pull down "**Graph ⇒ Scatterplots**" is used to activate scatterplots. The points on the line are formed by entering the integers from –5 to 5 and using the MINITAB calculator to calculate the corresponding *Y* values. The *X* and *Y* values are as follows.

X	–5	–4	–3	–2	–1	0	1	2	3	4	5
Y	–24	–20	–16	–12	–8	–4	0	4	8	12	16

The points are connected to get an idea of what the graph of the equation $Y = 4X - 4$ looks like.

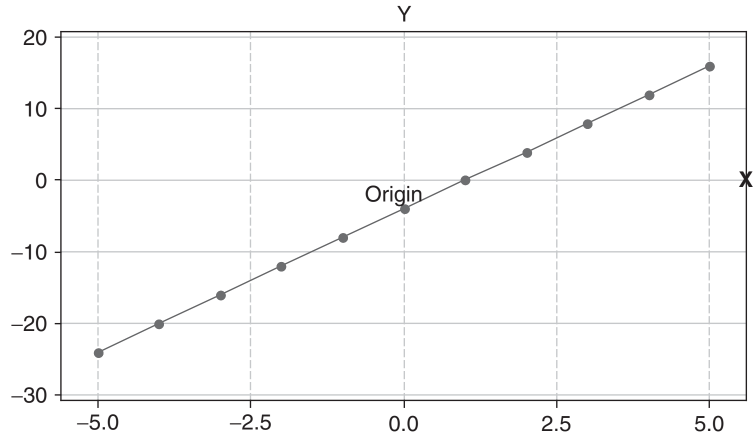

Fig. 1-5 A MINITAB graph of a linear function.

1.22 Graph the equation $Y = 2X^2 - 3X - 9$ using EXCEL.

SOLUTION

EXCEL is used to build Table 1.4 that gives the Y values for X values equal to −5, −4, . . . , 5. The expression =2*B1^2-3*B1-9 is entered into cell B2 and a click-and-drag is performed from B2 to L2. The chart wizard, in EXCEL, is used to form the graph shown in Fig. 1-6. The function is called a *quadratic function*. The *roots* (points where the graph cuts the x axis) of this quadratic function are at $X = 3$ and one is between −2 and −1. Clicking the chart wizard in EXCEL shows the many different charts that can be formed using EXCEL. Notice that the graph for this quadratic function goes off to positive infinity as X gets larger positive as well as larger negative. Also note that the graph has its lowest value when X is between 0 and 1.

Table 1.4 EXCEL-generated values for a quadratic function

X	−5	−4	−3	−2	−1	0	1	2	3	4	5
Y	56	35	18	5	−4	−9	−10	−7	0	11	26

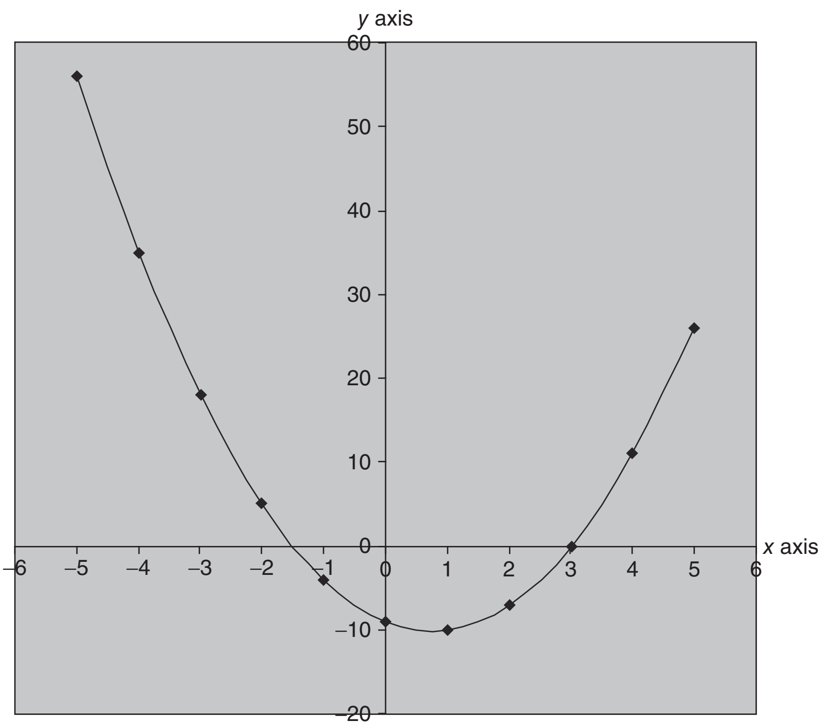

Fig. 1-6 An EXCEL plot of the curve called a parabola.

1.23 Table 1.5 shows the rise in new diabetes diagnoses from 1997 to 2005. Graph these data.

Table 1.5 Number of new diabetes diagnoses

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
Millions	0.88	0.90	1.01	1.10	1.20	1.25	1.28	1.36	1.41

SOLUTION

First method

The first plot is a *time series plot* and is shown in Fig. 1-7. It plots the number of new cases of diabetes for each year from 1997 to 2005. It shows that the number of new cases has increased each year over the time period covered.

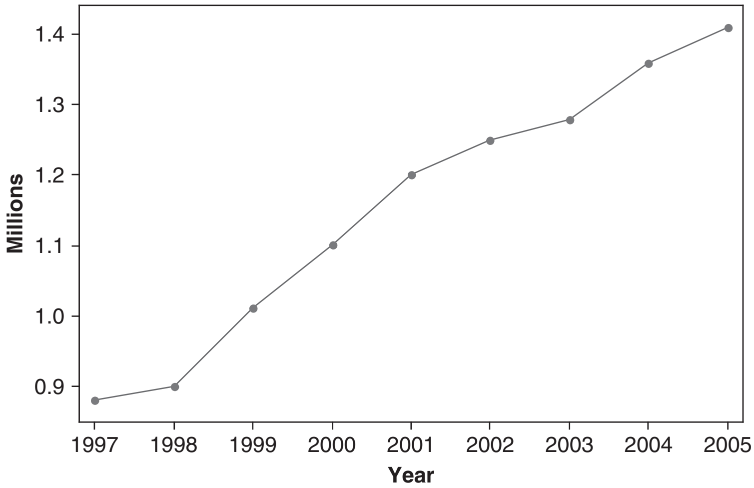

Fig. 1-7 MINITAB time series of new diagnoses of diabetes per year.

Second method

Figure 1-8 is called a *bar graph*, *bar chart*, or *bar diagram*. The width of the bars, which are all equal, have no significance in this case and can be made any convenient size as long as the bars do not overlap.

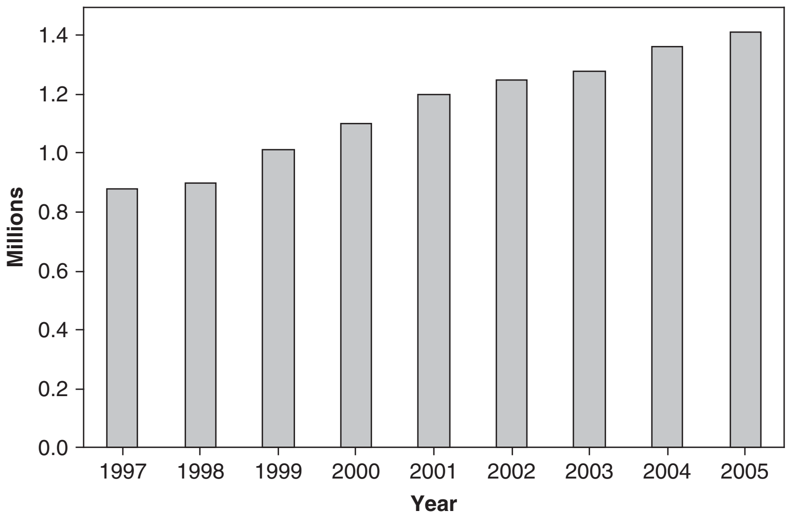

Fig. 1-8 MINITAB bar graph of new diagnoses of diabetes per year.

Third method

A bar chart with bars running horizontal rather than vertical is shown in Fig. 1-9.

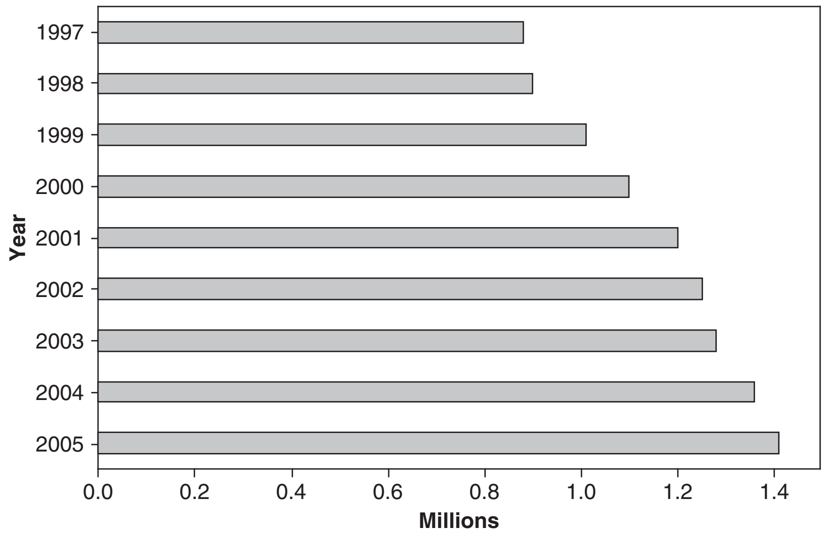

Fig. 1-9 MINITAB horizontal bar graph of new diagnoses of diabetes per year.

1.24 Graph the data of Problem 1.14 by using a time series from MINITAB, a clustered bar with three dimensional (3-D) visual effects from EXCEL, and a stacked bar with 3-D visual effects from EXCEL.

SOLUTION

The solutions are given in Figs. 1-10, 1-11, and 1-12.

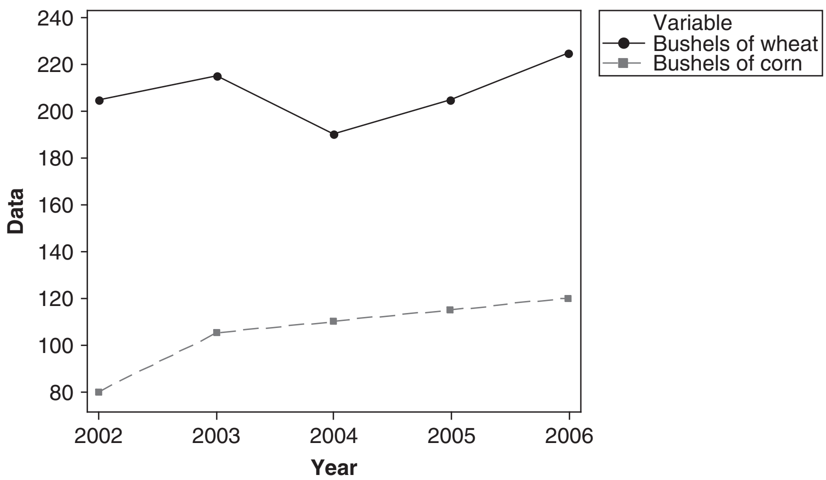

Fig. 1-10 MINITAB time series for wheat and corn production (2002–2006).

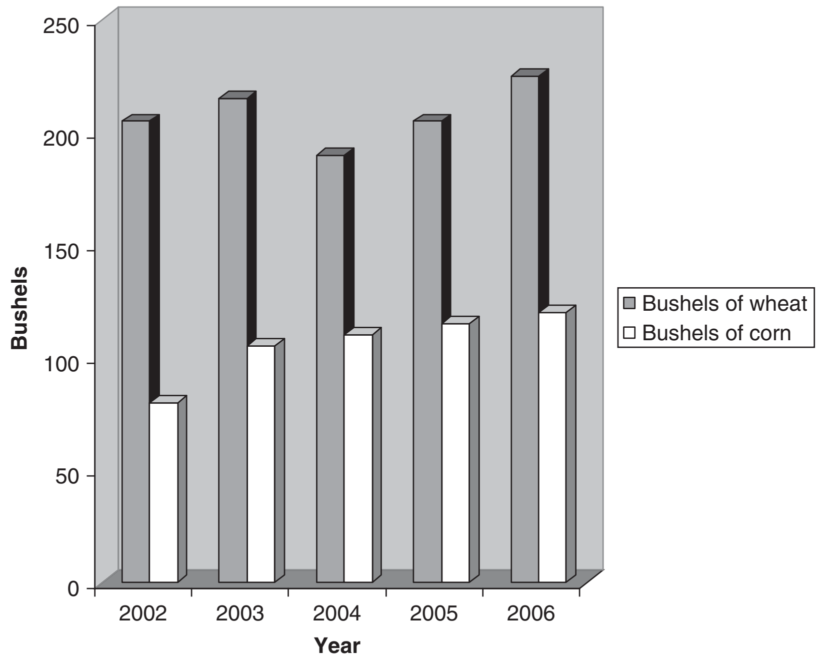

Fig. 1-11 EXCEL clustered bar with 3-D visual effects.

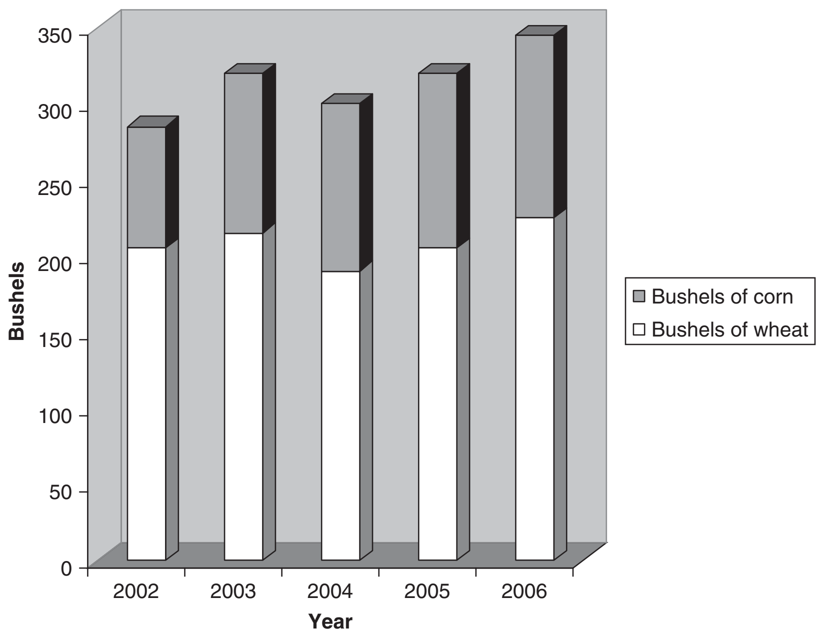

Fig. 1-12 EXCEL stacked bar with 3-D visual effects.

1.25 (*a*) Express the yearly number of bushels of wheat and corn from Table 1.2 of Problem 1.14 as percentages of the total annual production.

(*b*) Graph the percentages obtained in part (*a*).

SOLUTION

(*a*) For 2002, the percentage of wheat $= 205/(205+80) = 71.9\%$ and the percentage of corn $= 100\% - 71.9\% = 28.1\%$, etc. The percentages are shown in Table 1.6.

Table 1.6 Wheat and corn production from 2002 till 2006

Year	Wheat (%)	Corn (%)
2002	71.9	28.1
2003	67.2	32.8
2004	63.3	36.7
2005	64.1	35.9
2006	65.2	34.8

(*b*) The *100% stacked column* compares the percentage each value contributes to a total across categories (Fig. 1-13).

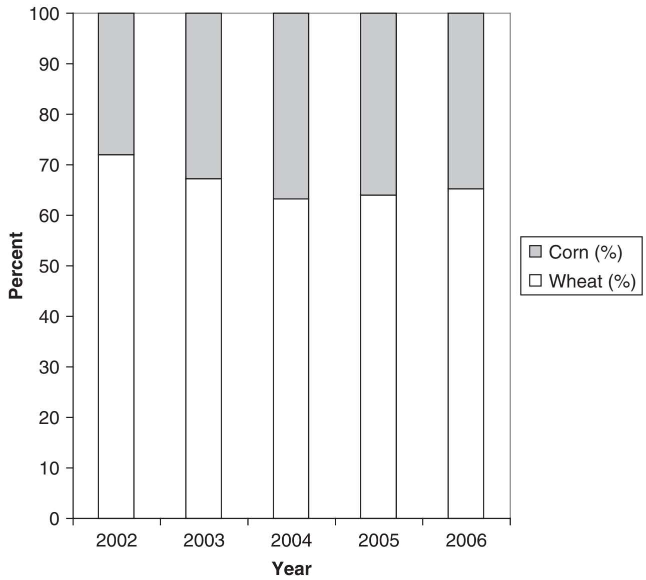

Fig. 1-13 EXCEL 100% stacked column.

1.26 In a recent *USA Today* snapshot entitled "Where the undergraduates are," it was reported that more than 17.5 million undergraduates attend more than 6400 schools in the USA. Table 1.7 gives the enrollment by type of school.

Table 1.7 Where the undergraduates are

Type of School	Percent
Public 2-year	43
Public 4-year	32
Private non-profit 4-year	15
Private 2- and 4-year	6
Private less than 4-year	3
Other	1

Construct a 3-D bar chart of the information in Table 1.7 using EXCEL and a bar chart using MINITAB.

SOLUTION

Figures 1-14 and 1-15 give 3-D bar charts of the data in Table 1.7.

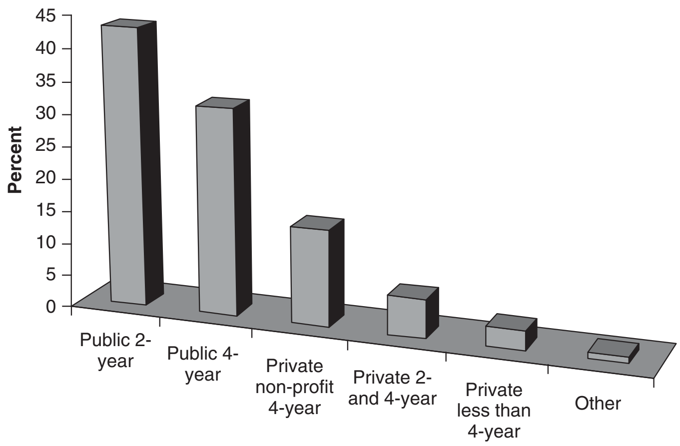

Fig. 1-14 EXCEL 3-D Bar chart of Table 1.7 information.

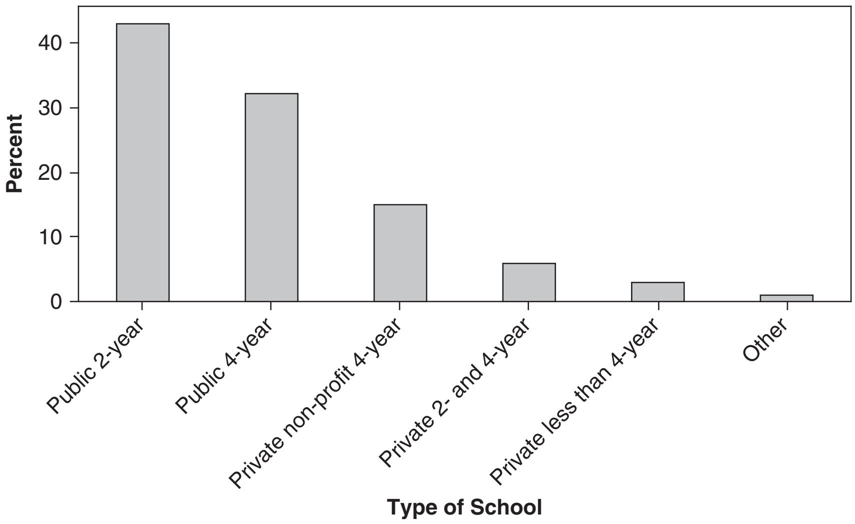

Fig. 1-15 MINITAB bar chart of Table 1.7 information.

1.27 Americans have an average of 2.8 working televisions at home. Table 1.8 gives the breakdown. Use EXCEL to display the information in Table 1.8 with a pie chart.

Table 1.8 Televisions per household

Televisions	Percent
None	2
One	15
Two	29
Three	26
Four	16
Five +	12

SOLUTION

Figure 1-16 gives the EXCEL pie chart for the information in Table 1.8.

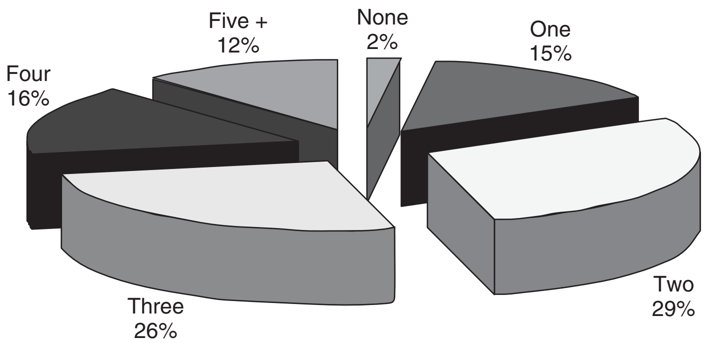

Fig. 1-16 EXCEL pie chart of information in Table 1.8.

Equations

1.28 Solve each of the following equations:

(*a*) $4a - 20 = 8$ (*c*) $18 - 5b = 3(b + 8) + 10$

(*b*) $3X + 4 = 24 - 2X$ (*d*) $\dfrac{Y+2}{3} + 1 = \dfrac{Y}{2}$

SOLUTION

(*a*) Add 20 to both members: $4a - 20 + 20 = 8 + 20$, or $4a = 28$.

Divide both sides by 4: $4a/4 = 28/4$, and $a = 7$.

Check: $4(7) - 20 = 8$, $28 - 20 = 8$, and $8 = 8$.

(*b*) Subtract 4 from both members: $3X + 4 - 4 = 24 - 2X - 4$, or $3X = 20 - 2X$.

Add $2X$ to both sides: $3X + 2X = 20 - 2X + 2X$, or $5X = 20$.

Divide both sides by 5: $5X/5 = 20/5$, and $X = 4$.

Check: $3(4) + 4 = 24 - 2(4)$, $12 + 4 = 24 - 8$, and $16 = 16$.

The result can be obtained much more quickly by realizing that any term can be moved, or *transposed*, from one member of an equation to the other simply by changing its sign. Thus we can write

$$3X + 4 = 24 - 2X \qquad 3X + 2X = 24 - 4 \qquad 5X = 20 \qquad X = 4$$

(*c*) $18 - 5b = 3b + 24 + 10$, and $18 - 5b = 3b + 34$.

Transposing, $-5b - 3b = 34 - 18$, or $-8b = 16$.

Dividing by -8, $-8b/(-8) = 16/(-8)$, and $b = -2$.

Check: $18 - 5(-2) = 3(-2 + 8) + 10$, $18 + 10 = 3(6) + 10$, and $28 = 28$.

(*d*) First multiply both members by 6, the lowest common denominator.

$$6\left(\frac{Y+2}{3} + 1\right) = 6\left(\frac{Y}{2}\right) \qquad 6\left(\frac{Y+2}{3}\right) + 6(1) = \frac{6Y}{2} \qquad 2(Y+2) + 6 = 3Y$$

$$2Y + 4 + 6 = 3Y \qquad 2Y + 10 = 3Y \qquad 10 = 3Y - 2Y \qquad Y = 10$$

Check: $\dfrac{10+2}{3} + 1 = \dfrac{10}{2}$, $\dfrac{12}{3} + 1 = \dfrac{10}{2}$, $4 + 1 = 5$, and $5 = 5$.

1.29 Solve each of the following sets of simultaneous equations:

(*a*) $3a - 2b = 11$, $5a + 7b = 39$ (*b*) $5X + 14Y = 78$, $7X + 3Y = -7$ (*c*) $3a + 2b + 5c = 15$, $7a - 3b + 2c = 52$, $5a + b - 4c = 2$

SOLUTION

(*a*) Multiply the first equation by 7: $21a - 14b = 77$ (*1*)
Multiply the second equation by 2: $10a + 14b = 78$ (*2*)
Add: $31a = 155$
Divide by 31: $a = 5$

Note that by multiplying each of the given equations by suitable numbers, we are able to write two *equivalent equations*, (*1*) and (*2*), in which the coefficients of the unknown b are numerically equal. Then by addition we are able to *eliminate* the unknown b and thus find a.

Substitute $a = 5$ in the first equation: $3(5) - 2b = 11$, $-2b = -4$, and $b = 2$. Thus $a = 5$ and $b = 2$.

Check: $3(5) - 2(2) = 11$, $15 - 4 = 11$, and $11 = 11$; $5(5) + 7(2) = 39$, $25 + 14 = 39$, and $39 = 39$.

(*b*) Multiply the first equation by 3: $15X + 42Y = 234$ (*3*)
Mutiply the second equation by -14: $-98X - 42Y = 98$ (*4*)
Add: $-83X = 332$
Divide by -83: $X = -4$

Substitute $X = -4$ in the first equation: $5(-4) + 14Y = 78$, $14Y = 98$, and $Y = 7$.

Thus $X = -4$ and $Y = 7$.

Check: $5(-4) + 14(7) = 78$, $-20 + 98 = 78$, and $78 = 78$; $7(-4) + 3(7) = -7$, $-28 + 21 = -7$, and $-7 = -7$.

(*c*) Multiply the first equation by 2: $6a + 4b + 10c = 30$
Multiply the second equation by -5: $-35a + 15b - 10c = -260$
Add: $-29a + 19b = -230$ (*5*)

Multiply the second equation by 2: $14a - 6b + 4c = 104$
Repeat the third equation: $5a + b - 4c = 2$
Add: $19a - 5b = 106$ (*6*)

We have thus eliminated c and are left with two equations, (*5*) and (*6*), to be solved simultaneously for a and b.

Multiply equation (*5*) by 5: $-145a + 95b = -1150$
Multiply equation (*6*) by 19: $361a - 95b = 2014$
Add: $216a = 864$
Divide by 216: $a = 4$

Substituting $a = 4$ in equation (*5*) or (*6*), we find that $b = -6$.
Substituting $a = 4$ and $b = -6$ in any of the given equations, we obtain $c = 3$.
Thus $a = 4$, $b = -6$, and $c = 3$.
Check: $3(4) + 2(-6) + 5(3) = 15$, and $15 = 15$; $7(4) - 3(-6) + 2(3) = 52$, and $52 = 52$; $5(4) + (-6) - 4(3) = 2$, and $2 = 2$.

Inequalities

1.30 Express in words the meaning of each of the following:

(*a*) $N > 30$

(*b*) $X \le 12$

(*c*) $0 < p \le 1$

(*d*) $\mu - 2t < X < \mu + 2t$

SOLUTION

(*a*) N is greater than 30.

(*b*) X is less than or equal to 12.

(*c*) p is greater than 0 but less than or equal to 1.

(*d*) X is greater than $\mu - 2t$ but less than $\mu + 2t$.

1.31 Translate the following into symbols:

(*a*) The variable X has values between 2 and 5 inclusive.

(*b*) The arithmetic mean $\overline{X}$ is greater than 28.42 but less than 31.56.

(*c*) m is a positive number less than or equal to 10.

(*d*) P is a nonnegative number.

SOLUTION

(*a*) $2 \le X \le 5$

(*b*) $28.42 \le \overline{X} < 31.56$

(*c*) $0 < m \le 10$

(*d*) $P \ge 0$

1.32 Using inequality symbols, arrange the numbers 3.42, −0.6, −2.1, 1.45, and −3 in (*a*) increasing and (*b*) decreasing order of magnitude.

SOLUTION

(*a*) $-3 < -2.1 < -0.6 < 1.45 < 3.42$

(*b*) $3.42 > 1.45 > -0.6 > -2.1 > -3$

Note that when the numbers are plotted as points on a line (see Problem 1.18), they increase from left to right.

1.33 In each of the following, find a corresponding inequality for X (i.e., solve each inequality for X):

(*a*) $2X < 6$

(*b*) $3X - 8 \ge 4$

(*c*) $6 - 4X < -2$

(*d*) $-3 < \dfrac{X-5}{2} < 3$

(*e*) $-1 \le \dfrac{3-2X}{5} \le 7$

SOLUTION

(*a*) Divide both sides by 2 to obtain $X < 3$.

(*b*) Adding 8 to both sides, $3X \ge 12$; dividing both sides by 3, $X \ge 4$.

(*c*) Adding −6 to both sides, $-4X < -8$; dividing both sides by −4, $X > 2$. Note that, as in equations, we can transpose a term from one side of an inequality to the other simply by changing the sign of the term; from part (*b*), for example, $3X \ge 8 + 4$.

(*d*) Multiplying by 2, $-6 < X - 5 < 6$; adding 5, $-1 < X < 11$.

(*e*) Multiplying by 5, $-5 \le 3 - 2X \le 35$; adding −3, $-8 \le -2X \le 32$; dividing by −2, $4 \ge X \ge -16$, or $-16 \le X \le 4$.

Logarithms and Properties of Logarithms

1.34 Use the definition of $y = \log_b x$ to find the following logarithms and then use EXCEL to verify your answers. (Note that $y = \log_b x$ means that $b^y = x$.)

(*a*) Find the log to the base 2 of 32.

(*b*) Find the log to the base 4 of 64.

(*c*) Find the log to the base 6 of 216.

(*d*) Find the log to the base 8 of 4096.

(*e*) Find the log to the base 10 of 10,000.

SOLUTION

(*a*) 5; (*b*) 3; (*c*) 3; (*d*) 4; (*e*) 4.
The EXCEL expression =LOG(32,2) gives 5, =LOG(64,4) gives 3, =LOG(216,6) gives 3, =LOG(4096,8) gives 4, and =LOG(10000,10) gives 4.

1.35 Using the properties of logarithms, re-express the following as sums and differences of logarithms.

(*a*) $\ln\left(\frac{x^2y^3z}{ab}\right)$

(*b*) $\log\left(\frac{a^2b^3c}{yz}\right)$

Using the properties of logarithms, re-express the following as a single logarithm.

(*c*) $\ln(5) + \ln(10) - 2\ln(5)$ (*d*) $2\log(5) - 3\log(5) + 5\log(5)$

SOLUTION

(*a*) $2\ln(x)+3\ln(y)+\ln(z)-\ln(a)-\ln(b)$

(*b*) $2\log(a)+3\log(b)+\log(c)-\log(y)-\log(z)$

(*c*) $\ln(2)$

(*d*) $\log(625)$

1.36 Make a plot of $y = \ln(x)$ using SAS and SPSS.

SOLUTION

The solutions are shown in Figures 1-17 and 1-18.
A plot of the curve $y = \ln(x)$ is shown in Figures 1-17 and 1-18. As x gets close to 0 the values for $\ln(x)$ get closer and closer to $-\infty$. As x gets larger and larger, the values for $\ln(x)$ get closer to $+\infty$.

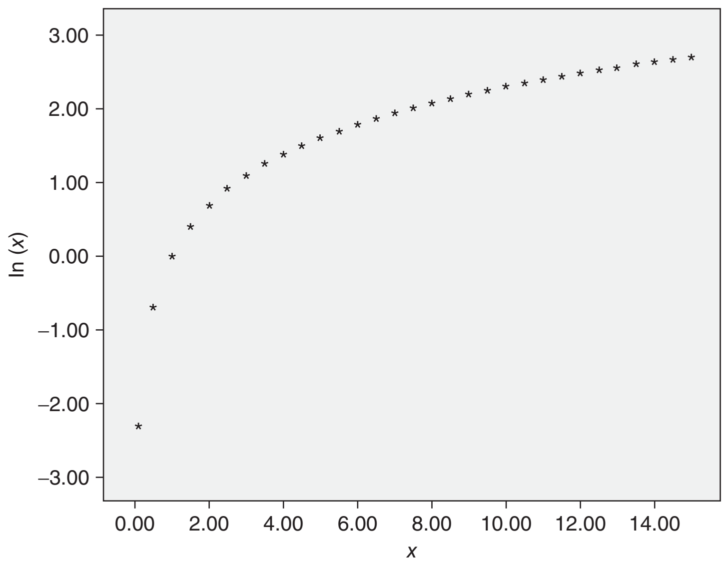

Fig. 1-17 SPSS plot of $y = \ln(x)$.

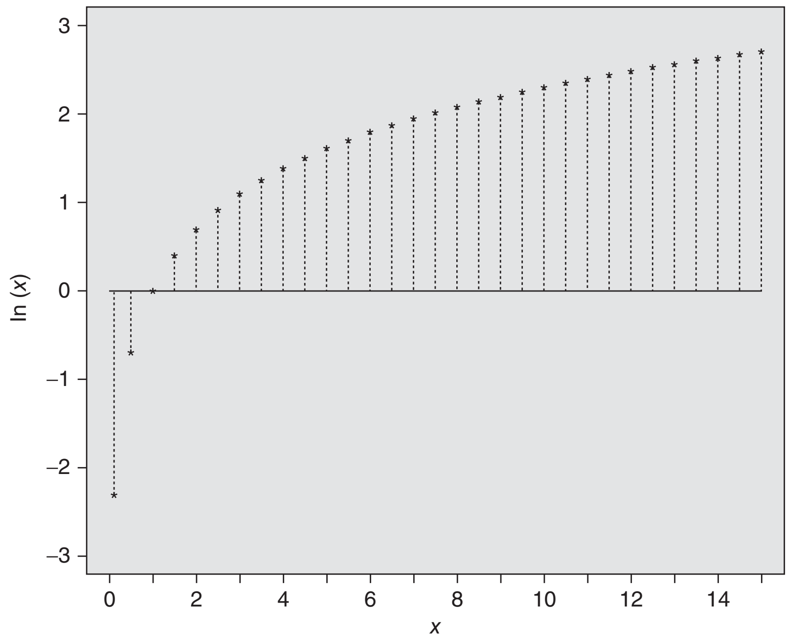

Fig. 1-18 SAS plot of $y = \ln(x)$.

Logarithmic Equations

1.37 Solve the logarithmic equation $\ln(x) = 10$.

SOLUTION

Using the definition of logarithm, $x = e^{10} = 22026.47$. As a check, we take the natural log of 22026.47 and get 10.00000019.

1.38 Solve the logarithmic equation $\log(x+2)+\log(x-2)=\log(5)$.

SOLUTION

The left-hand side may be written as $\log[(x+2)(x-2)]$. We have the equation $\log(x+2)(x-2)=\log(5)$ from which $(x+2)(x-2)=(5)$. From this follows the equation $x^2-4=5$ or $x^2=9$ or $x=-3$ or 3. When these values are checked in the original equation, $x=-3$ must be discarded as a solution because the log is not defined for negative values. When $x = 3$ is checked, we have $\log(5) + \log(1) = \log(5)$ since $\log(1) = 0$.

1.39 Solve the logarithmic equation $\log(a+4)-\log(a-2)=1$.

SOLUTION

The equation may be rewritten as $\log((a+4)/(a-2))=1$. Applying the definition of a logarithm, we have $(a+4)/(a-2)=10^1$ or $a+4=10a-20$. Solving for a, $a=24/9=2.6$ (with the 6 repeated). Checking the value 2.6667 into the original equation, we have $0.8239 - (-0.1761) = 1$. The only solution is 2.6667.

1.40 Solve the logarithmic equation $\ln(x^2)-1=0$.

SOLUTION

The equation may be factored as $[\ln(x)+1][\ln(x)-1]=0$. Setting the factor $\ln(x)+1=0$, we have $\ln(x)=-1$ or $x=e^{-1}=0.3678$. Setting the factor $\ln(x)-1=0$, we have $\ln(x)=1$ or $x=e^1=2.7183$. Both values are solutions to the equation.

1.41 Solve the following equation for x: $2\log(x+1)-3\log(x+1)=2$.

SOLUTION

The equation may be rewritten as $\log[(x+1)^2/(x+1)^3]=2$ or $\log[1/(x+1)]=2$ or $\log(1)-\log(x+1)=2$ or $0-\log(x+1)=2$ or $\log(x+1)=-2$ or $x+1=10^{-2}$ or $x = -0.99$. Substituting into the original equation, we find $2 \log(0.01) - 3 \log(0.01) = 2$. Therefore it checks.

1.42 The software package MAPLE may be used to solve logarithmic equations that are not easy to solve by hand. Solve the following equation using MAPLE:

$$\log(x+2) - \ln(x^2) = 4$$

SOLUTION

The MAPLE command to solve the equation is "solve $(\log10(x+2)-\ln(x\hat{}2)=4)$;" The solution is given as –0.154594.

Note that MAPLE uses log10 for the common logarithm.

To check and see if the solution is correct, we have log(1.845406) – ln(0.023899) which equals 4.00001059.

1.43 EXCEL may be used to solve logarithmic equations also. Solve the following logarithmic equation using EXCEL: $\log(x+4)+\ln(x+5)=1$.

SOLUTION

Figure 1.19 gives the EXCEL worksheet.

The iterative technique shown in the figure may be used. The upper half locates the root of $\log(x+4)+\ln(x+5)-1$ between –3 and –2. The lower half locates the root between –2.7 and –2.6. This process may be continued to give the root to any desired accuracy. Click-and-drag techniques may be used in doing the technique.

–3	–0.30685	LOG10(A1+4)+LN(A1+5)$^{-1}$
–2	0.399642	
–1	0.863416	
0	1.211498	
1	1.490729	
2	1.724061	
3	1.92454	
4	2.100315	
5	2.256828	
–3	–0.30685	LOG10(A11+4)+LN(A11+5)$^{-1}$
–2.9	–0.21667	
–2.8	–0.13236	
–2.7	–0.05315	
–2.6	0.021597	
–2.5	0.092382	
–2.4	0.159631	
–2.3	0.223701	
–2.2	0.284892	
–2.1	0.343464	
–2	0.399642	

Fig. 1-19 EXCEL worksheet for Problem 1.43.

1.44 Solve the equation in Problem 1.43 using the software package MAPLE.

SOLUTION

The MAPLE command "**> solve(log10(x + 4) + ln(x + 5) = 1);**" gives the solution –2.62947285. Compare this with that obtained in Problem 1.43.

SUPPLEMENTARY PROBLEMS

Variables

1.45 State which of the following represent discrete data and which represent continuous data:

(*a*) Number of inches of rainfall in a city during various months of the year

(*b*) Speed of an automobile in miles per hour (*c*) Number of \$20 bills circulating in the United States at any time

(*d*) Total value of shares sold each day in the stock market

(*e*) Student enrollment in a university over a number of years

1.46 Give the domain of each of the following variables and state whether the variables are continuous or discrete:

(*a*) Number W of bushels of wheat produced per acre on a farm over a number of years

(*b*) Number N of individuals in a family

(*c*) Marital status of an individual

(*d*) Time T of flight of a missile

(*e*) Number P of petals on a flower

Rounding of Data, Scientific Notation, and Significant Figures

1.47 Round each of the following numbers to the indicated accuracy:

(*a*) 3256	nearest hundred	(*f*) 3,502,378	nearest million
(*b*) 5.781	nearest tenth	(*g*) 148.475	nearest unit
(*c*) 0.0045	nearest thousandth	(*h*) 0.000098501	nearest millionth
(*d*) 46.7385	nearest hundredth	(*i*) 2184.73	nearest ten
(*e*) 125.9995	two decimal places	(*j*) 43.87500	nearest hundredth

1.48 Express each number without using powers of 10.

(*a*) 132.5×10^4

(*b*) 418.72×10^{-5}

(*c*) 280×10^{-7}

(*d*) 7300×10^6

(*e*) 3.487×10^{-4}

(*f*) 0.0001850×10^5

1.49 How many significant figures are in each of the following, assuming that the numbers are accurately recorded?

(*a*) 2.54 cm

(*b*) 0.004500 yd

(*c*) 3,510,000 bu

(*d*) 3.51 million bu

(*e*) 10.000100 ft

(*f*) 378 people

(*g*) 378 oz

(*h*) 4.50×10^{-3} km

(*i*) 500.8×10^5 kg

(*j*) 100.00 mi

1.50 What is the maximum error in each of the following measurements, assumed to be accurately recorded? Give the number of significant figures in each case.

(*a*) 7.20 million bu

(*b*) 0.00004835 cm

(*c*) 5280 ft

(*d*) 3.0×10^8 m

(*e*) 186,000 mi/sec

(*f*) 186 thousand mi/sec

1.51 Write each of the following numbers using the scientific notation. Assume that all figures are significant unless otherwise indicated.

(*a*) 0.000317

(*b*) 428,000,000 (four significant figures)

(*c*) 21,600.00

(*d*) 0.000009810

(*e*) 732 thousand

(*f*) 18.0 ten-thousandths

Computations

1.52 Show that (*a*) the product and (*b*) the quotient of the numbers 72.48 and 5.16, assumed to have four and three significant figures, respectively, cannot be accurate to more than three significant figures. Write the accurately recorded product and quotient.

1.53 Perform each indicated operation. Unless otherwise specified, assume that the numbers are accurately recorded.

(*a*) 0.36×781.4

(*b*) $\dfrac{873.00}{4.881}$

(*c*) $5.78 \times 2700 \times 16.00$

(*d*) $\dfrac{0.00480 \times 2300}{0.2084}$

(*e*) $\sqrt{120 \times 0.5386 \times 0.4614}$ (120 exact)

(*f*) $\dfrac{(416{,}000)(0.000187)}{\sqrt{73.84}}$

(*g*) $14.8641 + 4.48 - 8.168 + 0.36125$

(*h*) $4{,}173{,}000 - 170{,}264 + 1{,}820{,}470 - 78{,}320$ (numbers are respectively accurate to four, six, six, and five significant figures)

(*i*) $\sqrt{\dfrac{7(4.386)^2 - 3(6.47)^2}{6}}$ (3, 6, and 7 are exact)

(*j*) $4.120\sqrt{\dfrac{3.1416[(9.483)^2 - (5.075)^2]}{0.0001980}}$

1.54 Evaluate each of the following, given that $U = -2$, $V = \frac{1}{2}$, $W = 3$, $X = -4$, $Y = 9$, and $Z = \frac{1}{6}$, where all numbers are assumed to be exact.

(*a*) $4U + 6V - 2W$

(*b*) $\dfrac{XYZ}{UVW}$

(*c*) $\dfrac{2X - 3Y}{UW + XV}$

(*d*) $3(U - X)^2 + Y$

(*e*) $\sqrt{U^2 - 2UV + W}$

(*f*) $3X(4Y + 3Z) - 2Y(6X - 5Z) - 25$

(*g*) $\sqrt{\dfrac{(W-2)^2}{V} + \dfrac{(Y-5)^2}{Z}}$

(*h*) $\dfrac{X - 3}{\sqrt{(Y-4)^2 + (U+5)^2}}$

(*i*) $X^3 + 5X^2 - 6X - 8$

(*j*) $\dfrac{U - V}{\sqrt{U^2 + V^2}}[U^2V(W + X)]$

Functions, Tables, and Graphs

1.55 A variable Y is determined from a variable X according to the equation $Y = 10 - 4X$.

(*a*) Find Y when $X = -3, -2, -1, 0, 1, 2, 3, 4$, and 5, and show the results in a table.

(*b*) Find Y when $X = -2.4, -1.6, -0.8, 1.8, 2.7, 3.5$, and 4.6.

(*c*) If the dependence of Y on X is denoted by $Y = F(X)$, find $F(2.8)$, $F(-5)$, $F(\sqrt{2})$, and $F(-\pi)$.

(*d*) What value of X corresponds to $Y = -2, 6, -10, 1.6, 16, 0$, and 10?

(*e*) Express X explicitly as a function of Y.

1.56 If $Z = X^2 - Y^2$, find Z when (*a*) $X = -2$, $Y = 3$, and (*b*) $X = 1$, $Y = 5$. (*c*) If using the functional notation $Z = F(X, Y)$, find $F(-3, -1)$.

1.57 If $W = 3XZ - 4Y^2 + 2XY$, find W when (*a*) $X = 1$, $Y = -2$, $Z = 4$, and (*b*) $X = -5$, $Y = -2$, $Z = 0$. (*c*) If using the functional notation $W = F(X, Y, Z)$, find $F(3, 1, -2)$.

1.58 Locate on a rectangular coordinate system the points having coordinates (*a*) (3, 2), (*b*) (2, 3), (*c*) (–4, 4), (*d*) (4, –4), (*e*) (–3, –2), (*f*) (–2, –3), (*g*) (–4.5, 3), (*h*) (–1.2, –2.4), (*i*) (0, –3), and (*j*) (1.8, 0).